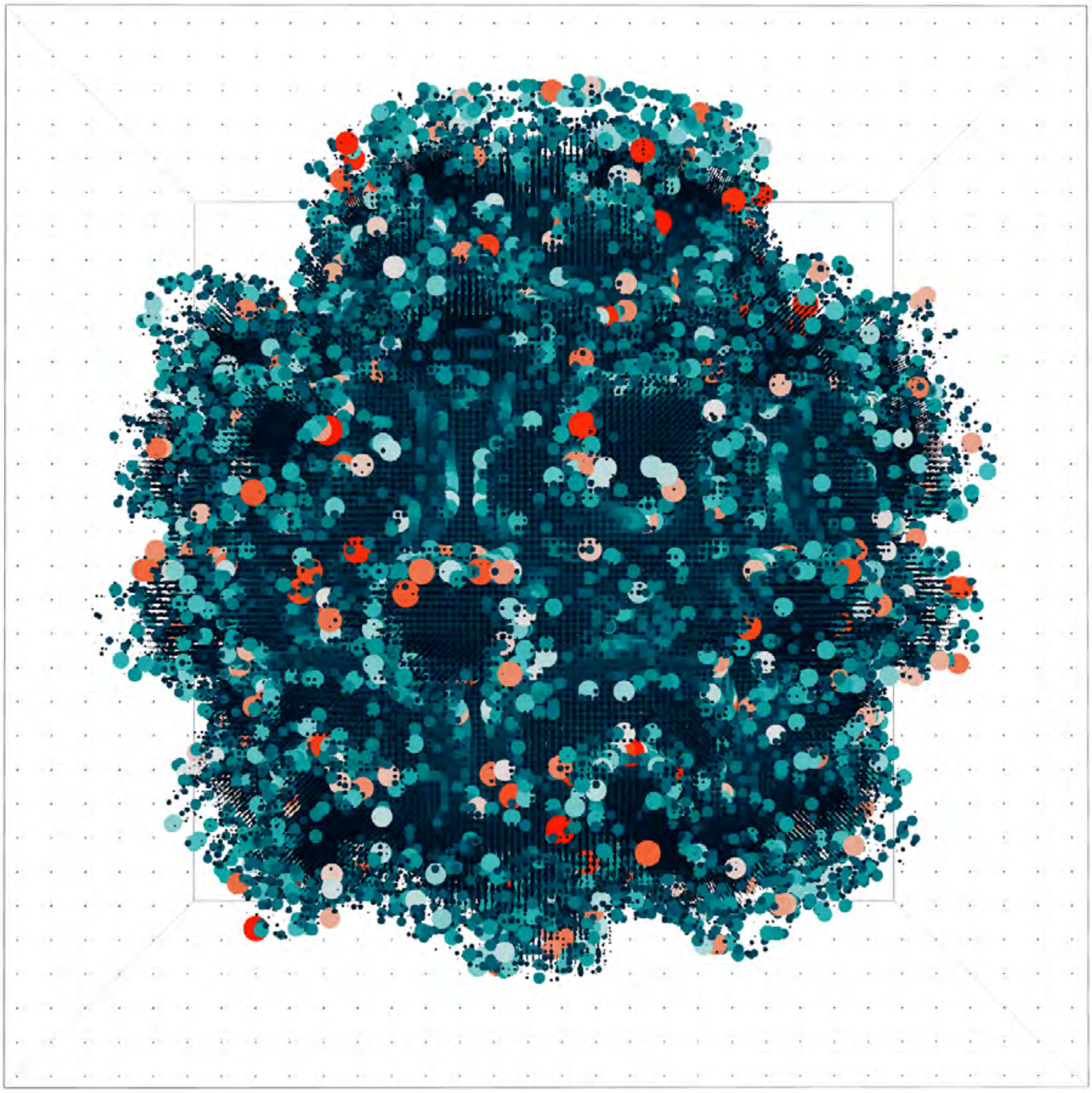
Julian Reil



Undecidable Systems

Exploring the inherent undecidable properties of reality and their implications on art and culture

Universität für Künstlerische und
Industrielle Gestaltung
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Exploring the inherent undecidable properties of reality and their implications on art and culture

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Abstract

This thesis aims to dissect and explore the broader topic of so-called "undecidable systems" and reflects on their possible implications concerning the paradigms of deterministic and indeterministic approaches to art and concepts of reality. It examines the logic of an undecidable system by showing examples from various perspectives and tries to identify the relations and mechanics of how art is intertwined with them due to its inherently undecidable properties which can be related to its abstract nature. It is an attempt to show that accumulating subjective experience, within the framework of art, philosophy and science, is a necessary and complimentary tool to give context to the search for knowledge by providing meaning and perspective.

Also, here is a little Mandelbulb for everyone.

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I wish you all the best.

Introduction

Looking into the abyss

This thesis might not even exist. And I do not mean because it might not have been written formally for any arbitrary reasons that could have affected me as the author. Regardless of its content, it might actually only exist relative to you reading it right now. Outside of the relation between you as the reader and the digital or physical manifestation of this thesis, there is no proof for it even being there. Before you start reading, its realness must in fact be something categorized as inherently undecided.

The same can be said about any kind of interaction with anything our mind has ever perceived. Art and what it might mean in the context of experiencing it, is itself also an undecidable system, as its properties are inherited from what we conclude from our own awareness relative to each other object we interact with. This perspective is the result of combined efforts of the scientific, philosophical and artistic developments of the 20th century and the many adventurers that were not afraid to venture into the unknown. Their efforts had a significant impact on our general perception of the world we live in, how we perceive ourselves in relation to other entities in it, and as a part of the cosmos that surrounds us. We have come to a point where, with the help of abstractive methods, we are slowly lifting the veil of deterministic and absolute variables and constants having long been believed as forming the foundation of reality itself and are witnessing the transformation of long standing concepts into something far more mysterious than we have ever imagined.

Many questions that we so far have reflected on and explored in an artistic and philosophical way may be inherently bound to an indeterministic truth that not only stems from the axioms of long forgotten times, but a truly and intrinsically undecidable nature that lies beneath the foundation of the laws of physics. It roots deep downwards the very bottom of existence itself and thus is inseparably interwoven with our perception of "what is".

As art is always a common denominator of the cultural process within its time and space, it accompanies us on the journey of reflecting and understanding ourselves and the realities that we live in. It is thus also unwillingly and indubitably affected by only ever being something that, although without strict confines, exists in relation to our perception of our own existence. It is a mirror to our subconscious and a telescope to our future while being a tool forged in our endless curiosity and pursuit of self-awareness and knowledge. It is an abstraction of reality.

In this paper I will show that the undecidable aspects of nature itself are afflicting their implications on art and culture. Sometimes we walk past the things that lie in our path in ignorance, without ever looking down, because we presume to have a solid understanding of the underworks that act as the pillars of our beliefs. But at the same time, one should not be afraid to stop once in a while and explore the abyss that lies directly beneath us. It is only knowledge and beauty that awaits.

Research questions

1. How do inherent undecidable properties in art and artistic practice reflect the complexities of human experience and perception of reality?

2. Which specific undecidable systems can be identified through an analysis of various artistic approaches and their underlying scientific and philosophical principles?

The deep dive

Talking robots to death

HARRY MUDD: 'Now listen to this carefully, Norman: I AM LYING!'

ROBOT NORMAN: 'You say you are lying, but if everything you say is a lie then you are telling the truth, but you cannot tell the truth because everything you say is a lie, but... you lie, you tell the truth, but you cannot for you I... Illogical! Illogical! Please explain! You are human! Only humans can explain their behavior! Please explain!'

CAPTAIN KIRK: (sarcastic) 'I am not programmed to respond in that area!'

ROBOT NORMAN explodes. (Roddenberry et al., 1967)

I am programmed to respond in that area instead: When we talk about undecidable systems, it is important to understand that, even though we might have never heard of them before, we come to find them in places we never expected them to be in the first place. They keep out of sight until we start asking the right questions. They leave us unbothered unless our curiosity leads us down a path where we want to know, where we want to find an answer to complement our thirst for knowledge. But can we actually know anything we seek to know? Wanting to know if we actually *can* know is a state of mind relatively new in the field of science and cognition, hence the term "ignoramus et ignorabimus". It is a Latin maxim that translates to "we do not know, we will not know". It is a synonym for the idea that scientific knowledge is and forever will be limited, which was introduced by Emil du Bois-Reymond, a German physician, in 1872 *(Anacker & Moro, 2016)*.

Only the bold and curious might set out on a journey to prove otherwise. One such individual was David Hilbert, one of the most influential German mathematicians of the 19th and early 20th century, who took a stance against the "ignorabimus" by saying: "We must know, we will know!" *(Morris, 2006)*. So what, exactly, *is* an undecidable system? Its general definition goes as follows:

> *The non-existence of an algorithm or the impossibility of proving or disproving a statement within a formal system. (Encyclopedia of Mathematics, n. d.).*

This means that an undecidable system is something that denies itself to be either verified or falsified within its given formal ruleset. This does not mean it cannot be true or false, it simply means that it cannot be proven that it is either of it. While there are probably uncountable formal systems that can be classified as undecided, the term itself stems from arithmetic decision problems from the early 20th century, where prominent mathematicians such as Kurt Gödel thought about formal "truth" in a sense of deducting logical statements from axioms inside a formal system referenced to in the following as "P" :

> *Thus Gödel's two great theorems are theorems about his calculus P: they assert the 'unprovability' within P of certain well-formed formulae of P (on the assumption that P is 'w-consistent' or 'consistent' respectively). Of course the interest to the learned world of the calculus P is that it can be regarded as representing a deductive system for arithmetic in which, therefore, there are undecidable arithmetical propositions. Though Gödel's formal proofs apply only to P, he indicates how similar proofs would apply to any calculus satisfying two very general conditions (p. 62), conditions so general that any calculus capable of expressing arithmetic can hardly fail to satisfy them. So this paper of Gödel's proclaimed the thesis, which has been clarified and confirmed by the work of subsequent metamathematicians, that no calculus can be devised in such a way that every arithmetical proposition is represented in it by a formula which is either 'provable' or 'disprovable' within the calculus, and therefore that any deductive system whatever for arithmetic will have the general syntactical characteristic of not containing either a proof or a disproof of every arithmetical proposition. This syntactical fact about*

> *arithmetic is sometimes described by saying that arithmetic, in its very nature, is incomplete. (Braithwaite,1992, p. 32)*

While the above statement seems purely mathematical in nature, its underlying concepts can be applied to many other systems outside of mathematics itself. Logic and reality in many cases bow to the implications of these ideas and lead to conclusions not satisfying for simple binary answers like "yes" or "no", which is what this paper is all about. But let us not get ahead of ourselves.

Most of the time we don't even need to think about undecidable systems in general. They seem to exist in their own realm outside of practicality and are most often not invited to the delights of our already complicated trains of thought. We want to keep things tidy in our minds. They don't seem to affect us, just like the things that happen in the vastness of the cosmos don't seem to affect our everyday life and its routines. Those kind of things are often highly unrelatable to us. Also, if considered, their implications mostly do not tend to influence the decisions we make on a daily basis, although we are probably only and understandably conveniently ignorant *(Heffernan, 2011)*. But as is true for so much that still lies in the dark, it is exactly these obscure things that might turn out to be hidden diamonds at the very core of the fabric of our realities themself.

People like Kurt Gödel often also found ways to express their mathematical conclusions into philosophical approaches or vice versa. It seems that there has to be some sort of sense for us as human beings in order to construct meaning for any formal system that can be "thought" into existence. Hau Wang remembers a conversation with Gödel regarding the "perfection of the world" and the "futility of another life that does not remember the previous one" *(Wang, 2001, p. 317)*:

> *Our total reality and total existence are beautiful and meaningful - this is also a Leibnizian thought. We should judge reality by the little which we truly know of it. Since that part which conceptually we know fully turns out to be so beautiful, the real world of which we know so little should also be beautiful. Life may be miserable for seventy years and happy for a million years: the short period of misery may even be necessary for the whole. (Wang, 2001, p. 317)*

I interpret Gödel's thought as follows: Even if we will never be capable of fully understanding the cosmos that surrounds us, even if some things might forever be in the dark and even if we do not find the answers we are looking for in our lifetimes, we should still be comforted by the fact that the things we can know about ourselves and reality are so immensely fascinating that it leaves enough room for deducting that everything that lies "beyond" us should also be at least as intriguing as the realms we are capable of understanding.

So in itself, an undecidable system might seem very elusive until we put our finger on it. And when we do, we quickly realize that it must be identified as a concept encoded in logic itself *(Meltzer, 1967)*. For the artificial intelligence named "Norman" in the original Star Trek Series, this posed a serious problem. An undecidable problem, in fact, which leads to self-destruction. But why? The underlying principle for this overly dramatic example in popular culture is inspired by the concept of a so called "logic bomb", which is one example for a conceptual undecidable system. It is also broadly known as the "liar paradox" which has been troubling philosophers for more than 2000 years and originates in Ancient Greece about 600 years B. C. *(Isalan, 2009)*. Its premise can be summarized as follows:

- The following statement is true. The preceding statement was false.

Norman the robot is a computer that operates on logical statements, which, just as with most of the computers we use today, can either be true or false - zeros or ones *(Balch, 2003)*. There are some exceptions to this, like the so-called *analytical engine* invented by the British mathematician Charles Babbage around 1834. The analytical engine is essentially a computer based purely on mechanical parts instead of binary transistors to calculate the results of linear equations *(Computer History Museum, n.d)*. However, this was more of a theoretical nature and a rather technical obscurity compared to the more practical computer we know today. Hypothetically, if a modern computer or a program is presented with a logically self-conflicting statement or operation akin to a liar's paradox, it should not be able to determine the outcome of that operation as either true or false; thereby defaulting into an endless loop of trying to determine the result of one statement referencing its contradictory counterpart *(Erdelsky, 1987)*. Applying this to the realms of software engineering and program analysis might result in a so-called "impossible program" which has been documented many times in computer theory:

> *A well-known piece of folk-lore among programmers holds that it is impossible to write a program which can examine any other program and tell, in every case, if it will terminate or get into a closed loop when it is run. (Strachey, 1965)*

Such programs will often result in *hanging executions (Lucas, 2021)*. In the realms of science fiction, this would mean either exploding (as in Star Trek), halting at some moment in time because of getting "stuck", or trying to solve the problem ad infinitum. Or would it? The answer might be non-verifiable or non-falsifiable. It might be in fact algorithmically unsolvable and thus undecidable *(Lucas, 2021)*. While this is a rather primitive example of something being undecidable, as with so many things, the more and closer someone looks, the more one comes to the realization that the idea of something being undecidable is not even particularly rare after all *(Poonen, 2014)*. Apart from the context of our everyday experiences, we often find ourselves in situations where we are trying to figure out what the answer to a given problem might be. Sometimes even the intricate answers to the questions we only ask ourselves in private are elusive at best and non-existent at worst. These are often the more profound ones, the same which let us stare into the collective abyss of doubt and desperation. They often act like a mirror and define who we think we are.

While these inner thoughts are more of a philosophical nature, they still feel as if they have some kind of deterministic undecidability attached to them in some cases. Of course, as human beings we enjoy the privilege of simply ignoring those thoughts, or we simply define answers that please us just enough to forget about the problem in the first place *(Heffernan, 2011)*. We also have invented tools to aid us in our quest of finding peace over the more undecidable matters of our realities: science, art, philosophy, and religion. While from a scientific point of view I would not consider religion a very good source for guidance for the most part, it is still true that many fundamental questions have been tried to be answered by its fantastic propositions. But as history has shown us throughout the past, it is often not wise to take a shortcut for understanding complicated things just to get a simple answer that satisfies our curiosity to a sufficient degree *(Dawes, 2016)*. If one seeks out to discover all the beauty life has to offer, the right way may turn out to be a path of perseverance and and intrigue rather than simplicity. As will be shown in this thesis, the same is true for undecidable systems.

An undecidable definition

Defining something is the necessary act of agreeing on certain properties that can then be summarized as the "definition" of an entity on a linguistic meta level. This is sometimes easier said than done. Take art, for example. What is art? Can art even be defined without the context of its materialistic manifestation, i. e. the actual "artwork"?

What really defines art, practicing art or even what the prerequisites are for something being branded as an work of art is notoriously hard to answer. As *The Oxford Handbook of Aesthetics* tries to demonstrate:

> *The project of defining art most commonly consists in the attempt to find necessary conditions and sufficient conditions for the truth of the statement that an item is an artwork. That is, the goal is normally to find a principle for classifying all artworks together while distinguishing them from all non-artworks. Sometimes the goal is set higher. Some look for a 'real' definition: that is, one in terms of necessary conditions that are jointly sufficient for being an artwork. Sometimes the aim is to identify a metaphysical essence that all artworks have in common. (Stecker, 2003, p. 136)*

When the common denominator is something like a "metaphysical essence", it becomes impossible to quantify it in a sense that it lends toward a satisfactory truth for every instance that definition tries to explore. That is what makes art as a concept so incredibly slippery when it comes to defining it. While there are certainly more conservative approaches for coming to a conclusion like the "conventionalist definition" or the "institutional definition" or the "traditional definition" (also considered the "aesthetic definition", just to name a few) *(Adajian, 2018)*.

Just to briefly illustrate how contradictory these definitions are towards each other, I will give you a glimpse into the corner stones of the ones mentioned above. I will keep it very brief, since I rather want to show their conflicting nature than debate every single one there is:

- Conventionalist Definition: Conventionalist definitions deny that art has essential connection to aesthetic properties, or to formal properties, or to expressive properties, or to any type of property taken by traditional definitions to be essential to art. Conventionalist definitions have been strongly influenced by the emergence, in the twentieth century, of artworks that seem to differ radically from all previous artworks. *(Adajian, 2018)*

- Institutional Definition: The groundwork for institutional definitions was laid by Arthur Danto, better known to non-philosophers as the long-time influential art critic for the *Nation*. Danto coined the term "artworld", by which he meant "an atmosphere of art theory." Danto's definition has been glossed as follows: something is a work of art if and only if (i) it has a subject (ii) about which it projects some attitude or point of view (has a style) (iii) by means of rhetorical ellipsis (usually metaphorical) which ellipsis engages audience participation in filling in what is missing, and (iv) where the work in question and the interpretations thereof require an art historical context (Danto, Carroll). *(Adajian 2018)*

- Traditional (mainly aesthetic) Definition: Traditional definitions take some function(s) or intended function(s) to be definitive of artworks. Here only aesthetic definitions, which connect art essentially with the aesthetic – aesthetic judgements, experience, or properties – will be considered. Different aesthetic definitions incorporate different views of aesthetic properties and judgments. *(Adajian, 2018)*

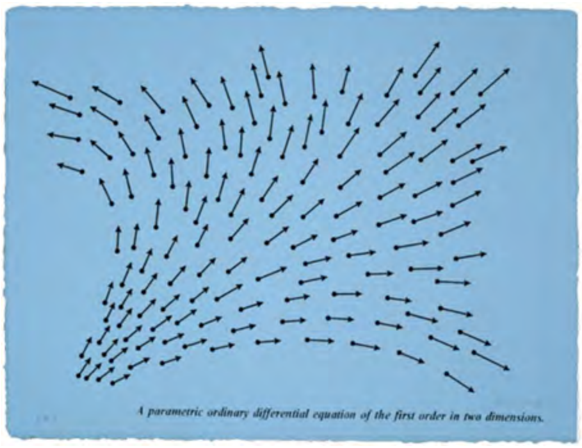
As complicated as it seems to come to a conclusion, we can still state that even art itself, while eluding any form of definition indefinitely and rightfully so, is always a representation of something in some kind. No matter what topic it (art) or the artist does reflect on, it is always an abstraction of of reality to some degree. And to have the freedom of choosing how far away the relation of the abstraction is relative to its original motive, is what I consider one of the true powers the system we call art has in its repertoire. It goes as far as being able to produce art without even producing art. Or as Bernar Venet, a renowned French conceptual artist puts it:

> *At a certain point I stopped producing art, because I thought that, you know, there is no need for me to just make paintings for the pleasure of making paintings to decorate the walls of collectors, to satisfy the museum, to satisfy my need of making art because otherwise I can not live. Ridiculous. No, no… Art is about giving a new definition about art. (NOWNESS, 2022)*

Art is as much about having a concept as it is in producing something within that framework. Bringing something into existence is a process of creation rather than a process of art per se; although it can serve the purpose of giving an otherwise mere concept a form and a body to explore for others. If we see art primarily as a conceptual process, one can also introduce any form of abstract elements to it while still not violating any form of definition within the system. Thus also very rational elements which are used to describe the world around us can be entities of artistic concepts, like science, or, more specifically, mathematics. Bringing a concept into life can be a process that does not invoke doing art, per se. It is much more the craft or the labor of creating that is obligatory for art to manifest. Once again, Bernar Venet:

> *[W]orking on art is terrible for me. I just don't like it. What I like is to have a concept, is to have an idea and to think: Oh my god, I want to do that. So I did it, I introduced rationality as much as possible. The use of language, mathematical language and I went as far as possible (….) When I started to make my mathematical paintings in 1966 I was not even sure it was going to be valid, to be accepted. My artist friends of my generation came and they thought that I was going crazy. They said: You know it's not art, it's mathematics. You see, there is nothing more abstract than a mathematical diagram. It is more abstract than a painting by Rothko. (NOWNESS, 2022)*

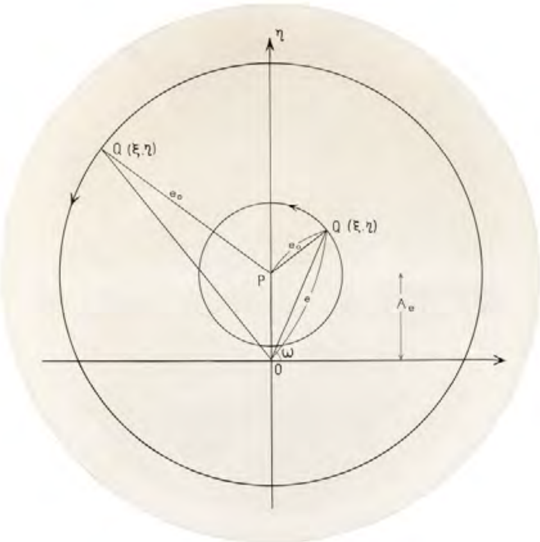
Figure 1, Artworks of Bernar Venet



(*A Parametric Ordinary Differential Equation of the First Order in Two Dimensions*, 2001)

(*Shaped Geometric Canvas*, 2024)

(*13 Acute Unequal Angles*, 2019)



(*Relations Between Eo and E for Nearly Circular Orbit*, 2021)

(*Gold Saturation With "We Determine Finitely"*, 2009)

Expressing artistic ideas through rational or mathematical language inherently brings in those abstract elements evolving from their respective axioms and adherent logical formulations. They thereby automatically expose many of their deterministic and indeterministic properties from which many, as we will explore in this paper, have intrinsic undecidable properties *(Chaitin et al., 2018)*.

They are, in fact, such a common occurrence, that there happen to be many definitions for them in all the different disciplines, like art or science. Most of them stem from a logical or mathematical perspective due to the ease of their reproducibility. So how could we approach defining what an undecidable problem or system really is? In the following, I will formulate one of the more popular approaches to this topic, based on what has already been demonstrated in this paper so far:

An undecidable problem is a form of decision problem for which it is proven that there can never ever be an arithmetic algorithm that correctly determines the answer (true or false) throughout infinity *(Encyclopedia of Mathematics, n.d.)*. While this seems very abstract at first, it can also be thought about as something we consider unsolvable due to deductive thinking. We are often able to articulate a logical or philosophical question which we know can never be answered due to its paradox nature.

One of these examples is "Russell's paradox", which is briefly illustrated below:

- We divide everything that exists (in the whole universe) into sets.
- So there is a set of all students, a set of all shoes, a set of all cars, a set of all cats etc..
- Then we consider that there is a set of all the sets that are not members of themselves.
- This set then has to be included in itself but only if at the same time it is not included in itself. (Irvine & Deutsch, 2020)

So can there be something that only exists if it does not exist? I remember an old joke, the origins of which escape me. It goes something like this:

If the creation of the cosmos is considered to be the biggest achievement of any omnipotent creator or god, then it must have also been the hardest effort to achieve for such a creature. The biggest handicap for such an endeavor would therefore be to do it while at the same time not existing a priori. Thus, such a being could only have created the universe if and only if it does not exist. Otherwise, if that creator did it while at the same time also existing, it would not have been his biggest achievement nor his hardest effort. In other words: If he indeed exists AND created everything, he must have been just lazy and did not give it his best shot.

There are many similar thought experiments that very quickly enter the realm of elaborate "dad jokes" or being "the smart kid" at parties. That is because for the unsuspecting mind, they seem more like a magic trick designed to stun and awe with intricate logic rather than being something very real. But indeed they are very real. And they have consequences.

While the implications of such ideas might seem insignificant at first, we have to make clear to ourselves that knowing we can not find a correct answer to an arbitrary question is not as trivial as it might seem at first glance. When it comes to science, this does not just mean that we are simply not able to solve these questions yet and will eventually come to a point where we will be able to. In the broader context of philosophy of science, this means that it might be scientifically or philosophically impossible to ever find the correct answer to these kinds of problems. It means that the undecidability of such matters is not due to the lack of our understanding, but it is rather internalized inside a formal system itself *(Encyclopedia of Mathematics, n.d.)*. And as I said before, there are many problems of this kind. But why does this even matter?

An undecidable paradox

As we have shown above, the term "undecidable system" seems to be closely related to the term "paradox". But one must not make the mistake to think that they are just interchangeable synonyms. While undecidable systems can certainly be paradox in nature, this does not mean that paradoxes just are undecidable systems and vice versa *(Svozil, 2008)*. To understand this, we have to take a closer look on what a paradox actually is and what it means in the context of undecidability.

So "paradox" can have different definitions in different contexts. A paradox is a statement that, while logically valid or true, seems to be self-contradictory. Depending on the context, it may contradict generally accepted principles and thereby conflict with our perception of reality (in philosophy), it may be a logical riddle/ problem based on self-reference (in mathematics), or it may be used in an ironical sense to create a complex narrative (as a storytelling mechanism), among other things *(Cantini & Bruni, 2021)*.

What does this mean in the context of undecidable systems? Not all undecidable systems are necessarily paradoxical. As stated before, many undecidable systems arise in the study of mathematical structures and algorithms, and do not involve any inherent paradoxes.

So in the context of undecidable systems, a paradox is a statement or situation that appears to be self-contradictory or absurd while nonetheless being logically valid. Many more paradoxes can arise when dealing with complex systems, especially those involving self-reference or self-referential statements *(Bolander, 2017)*. But that does not mean they are undecidable. In order for something to be undecidable, there has to be a valid method connected which would theoretically lead to a definitive answer *(Encyclopedia of Mathematics, n.d.)*. This is a distinguished difference and a criterion a paradox, on the other hand, does not have to fulfill at all. So what would an undecidable paradox actually look like?

As mentioned earlier, one famous example of a paradox in the context of undecidable systems is Russell's paradox in set theory. The paradox arises from considering the set of all sets that do not contain themselves. If such a set exists, it results in contradiction: If the set contains itself, then it should not contain itself, and if it does not contain itself, then it should contain itself. This contradiction shows that the set of all sets that do not contain themselves cannot exist, and demonstrates the limitations of naive set theory, although it is logically valid *(Irvine & Deutsch, 2020)*.

Another example of a paradox in undecidable systems, which we also briefly touched on before, is the *liar's paradox*, which involves a self-referential statement that says "this statement is false". If the statement is true, then it must be false, and if it is false, then it must be true. This creates a paradoxical situation where the statement cannot be consistently assigned a "truth value" *(Beall et al., 2016)*.

It can be solved by iteration though: The first time "this statement is false", the truth value is that the statement is "false". But if we go into a logical feedback loop it would mean for the second iteration of the same statement that the statement itself is false and thus turn the "truth value" of the statement to "true". This will result in an endless loop as we have seen before and while the operation itself is valid, it is at the same time undecidable concerning the truth value of the statement and paradoxical in the sense that there will never be a definitive answer, although there is a valid method.

So, while undecidable systems can sometimes lead to paradoxes, it is not a necessary characteristic of all undecidable systems. Also, not all paradoxes are ultimately undecidable systems. Just because something is not logically solvable does not prove that it is not decidable in the context of being theoretically but not practically solvable. In general, paradoxes in undecidable systems can be seen as highlighting the limitations of formal systems and logic, and can therefore influence or lead to new insights and interesting thought experiments in science, art, and culture.

Paradoxes are also often used in art to create interesting and thought-provoking works that challenge the viewer's assumptions and expectations of a particular topic. Some artists use paradoxes to create optical illusions or impossible shapes, while others use paradoxes to create conceptual works that play with ideas of perception, reality, and identity. This is true for more classical art as well as for contemporary media art of course.

One very famous and classical use of a certain kind of paradox in art is M. C. Escher's *Waterfall* lithograph, which depicts a continuous waterfall that appears to be flowing upwards *(Escher, 1961)*.

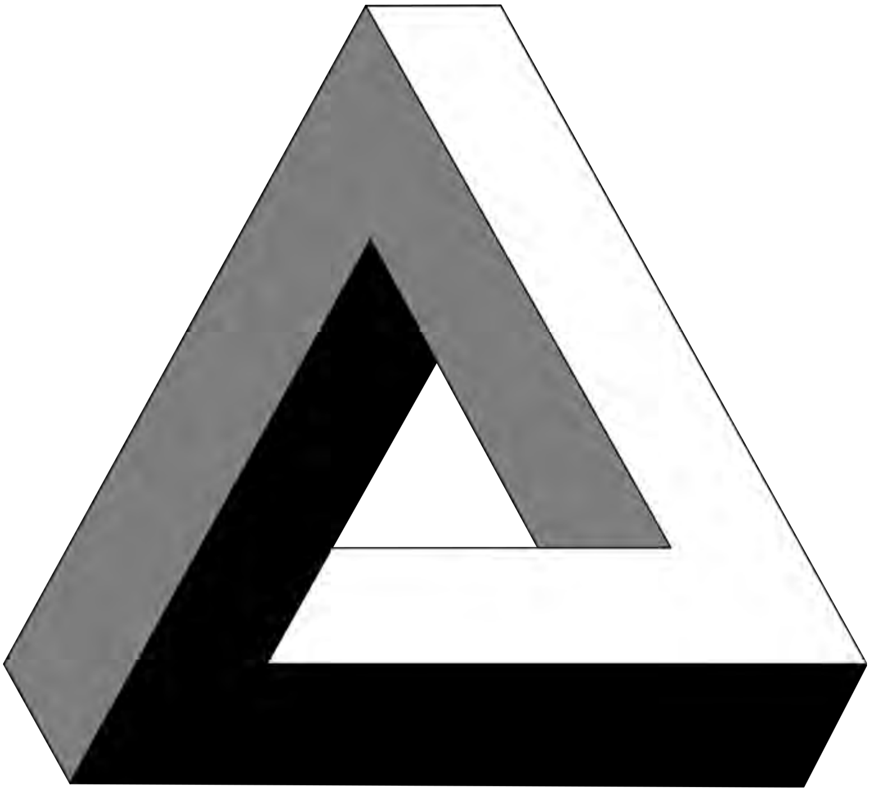
Figure 2, "Waterfall" by Maurits Cornelis Escher

(*Waterfall*, 1961)

The paradoxical nature of the image creates a sense of disorientation and confusion, thus inviting the viewer to question their perception of reality when trying to determine what is really going on there and how this effect comes into existence. Our brains do not simply give up and say "Well, there it is, a paradox.". No, our brain wants to solve the riddle, and our curiosity is what fuels it. Our brain "must know", as David Hilbert would put it *(Morris, 2006)*.

The way this particular illusion works is by making use of so-called 'conflicting proportions'. The channel through which the water flows features two Penrose triangles, a type of "impossible object" first conceived by Oscar Reutersvärd in 1934 *(Wikipedia contributors, 2021)*. This kind of triangle cannot exist as a physical object and can only be depicted as a perspective drawing. The Nobel prize laureate mathematician Roger Penrose and his father popularized the Penrose triangle in the 1950s, hence the name *(Penrose & Penrose, 1958)*. It is often referred to as "impossibility in its purest form", although the origin of that exact formulation remains unknown and can not directly be attributed to Penrose. Figure 3 shows a visualization of the Penrose triangle:

Figure 3, Penrose Triangle



(*Penrose Triangle*, 2024)

While this already hints at the mathematical nature of Escher's art, there is another detail which is often overlooked due to the centrality of the 'endless' waterfall as the piece's main feature: the two geometrical objects atop the tower. This was Escher's way of subliminally conveying his fascination with geometry and mathematics as artistic motives of his work *(Schattschneider, 2010)*.

Another very famous example of paradoxical art is the work of René Magritte, who often painted scenes containing contradictory or impossible elements. For example, his painting *The Treachery of Images* depicts a realistic image of a pipe with the caption *Ceci n'est pas une pipe* ("This is not a pipe"). The paradoxical nature of the image challenges the viewer's assumptions about the relationship between images and reality and invites them to consider the nature of representation and perception *(ReneMagritte.org, n.d.)*.

Figure 4, "The Treachery of Images" by René Magritte

(*The Treachery of Images*, 2024)

At first glance, the painting appears to be a simple representation of a pipe. However, the words beneath immediately hint towards the before mentioned duality. The painting is not a pipe, but a representation of a pipe, which is not the same as the real thing. This play on language and representation highlights the arbitrary and subjective nature of our perception of reality.

We often assume that a picture or representation is a faithful reproduction of the thing it depicts. However, Magritte's painting reminds us that images are constructed and mediated by language and culture, and that our perception of reality is always influenced by these factors. In this sense, "The Treachery of Images" can be seen as a critique of the traditional representational function of art. The painting challenges us to think about the limitations of representation and the ways in which images can both reflect and distort our perception of reality.

Lastly, paradoxes are also frequently used in more conceptual art forms, which focus on the idea or concept behind a work rather than its physical form. For example, in his 1961 work *One and Three Chairs*, artist Joseph Kosuth presented a chair in three different forms: as a physical chair, as a photograph of the same chair, and as a dictionary definition of the word "chair" *(Museum of Modern Art, n.d.)*.

Figure 5, "One and Three Chairs" by Joseph Kosuth

(*One And Three Chairs*, 2024)

The paradoxical nature of this work challenges the viewer's assumptions about what constitutes a chair and invites them to reconsider the relationship between language, perception, and representation. What does "a chair", or really anything, really mean? What does "mean" mean?

As Carlo Rovelli, a theoretical physicist who has contributed a lot to the modern physics of space and time, puts it in his book *Helgoland*:

> *A technical term for 'referring to something' in our mental processes is "intentionality." Intentionality is an important aspect of the notion of meaning and our whole mental life. There is a close relationship between what happens in thoughts and what happens 'outside' of thoughts: what thoughts mean. There is a close relationship between the word "cat" and a cat; between a road sign and what the road sign signifies.*

> *There seems to be nothing of any of this in the natural world. A physical event in itself 'means' nothing.*
>
> *If we are part of the physical world, this world of meanings must emerge from the physical world. How? What is the world of meanings, in purely physical terms? (Rovelli, 2021, pp. 166-167)*

We can conclude that art is often intentionally paradoxical, and thus, carefully crafted around paradoxical phenomena. This serves as a powerful tool for creating works that challenge the viewer's assumptions about the physical world and invite them to think deeply about the nature of reality, perception, identity, and their place within all these.

An undecidable method

As Arthur Miller argues in his book *Insights of Genius: Imagery and Creativity in Science and Art* it is artists and scientists alike who have always been on a quest to explain the world around us and/or expose a variety of possibilities in the realm of what we consciously perceive. We try to make sense of our surroundings and ourselves. We are constantly trying to express what we feel and how we perceive the surrounding world while also reinventing our tools and methods for understanding the complexity of our nature and realities themselves *(Miller, 2012)*.

As I see it, one can categorize art in general as an effort or a tool for reflecting on nearly anything they want. It is a looking glass with a mirror attached that can be used to underline or explore certain aspects of reality itself. It can be a means of expression for society and a catalyst for culture. Also, as we have seen previously, it can be defined by having no agreed upon definition at all. In doing so, it is freed of any constraints of method. It can use whatever is at its disposal for articulating itself. There is no need for any kind of restrictive system, for following any particular rule, for being reproducible in any way, having a certain format or even be understandable in a materialistic way.

"This world is but a canvas to our imagination." *(Thoreau, 1849)* as David Henry Thoreau put it. What he means is that art is best described as any effort someone takes in order to make their own ideas and dreams tangible for others, that we are also our own creators of the realities we live in. On the other end of the spectrum, we have the broad field of science. In some way it can be described as the obedient twin of art, as it also often acts as a foundation of bringing ideas or dreams to life, and thus, painting the canvas with our imagination. As the mathematician Stefan Banach put it: "Mathematics is the most beautiful and most powerful creation of the human spirit." *(Kaluza et al., 1996)*. But compared to art, science is bound to follow certain rulesets. It is its own inherent attribute to achieve a high level of conformity within its own scientific methods. This is what makes it valuable in a materialistic sense, reproducible and universally useful for whichever goal there is to achieve, or whichever question it seeks to answer *(Hepburn & Andersen, 2021)*.The common premise is: If we can formulate the right questions, we will eventually find the right answers. Science is a tool to understand the world around us. It contributes to our understanding of the universe and it speaks many languages - one of them being mathematics. One of the most famous works, considered to be amongst the most important pioneering works of the scientific method, is called *Il Saggiatore* by Galileo Galilei. His idea was to break with the paradigm of perceiving reality through the eyes of scholastic philosophy and instead apply the tools of science to solve the riddles that puzzle us since the beginning of mankind.

> *[The universe] cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth. (Galilei, 1623, p. 4)*

And in a way, one could argue, it still is only an idea of describing the world by giving it calculable attributes. Much like art, it must be an abstraction of reality itself because it describes something by encoding it into another logical system such as language. It means thinking about reality in numbers and abstractions rather than through the intuitive and tangible methods art has the freedom to inflict upon itself. There is a lot of faith in science that it should, can, and will prove itself invaluable when it comes to finding most of the answers buried deep inside the universe. Chronologically, we are just at the beginning of our journey as a species, and yet, we have already encountered and successfully taken many hurdles along the way. Or as Gene Roddenberry once said, "It isn't all over; everything has not been invented; the human adventure is just beginning." *(Sackett, n. d.)*

And yet, even in *Star Trek* it is culture and art that have prevailed throughout the scientific and technological evolution of our species. Even Spock, the most logical being known to mankind, is still bound by his species' rituals and practices which at times overrule all logic and suppression of emotions. Even for a fictional alien mind, art serves as a harbor to give context and meaning to all the knowledge one could accumulate over the eons of time. It stands at the very core of the experience of existing inside a universe which only slowly gives away the details of its inner workings. For every scientific breakthrough, there is an artistic and philosophical response mediating/facilitating human understanding. This is also what we as humans have always aimed for: to ask the right questions about our place within our culture. Or, as Hans Dieleman puts it:

> *In this [transdisciplinary] approach, reality is seen as complex and multilayered, and cannot be known only through the lens of modern science. Philosophy, art and subjective experience are essential and complementary ways of knowledge production that provide context to scientific knowledge, giving it meaning and perspective. To be able to engage in transdisciplinary hermeneutics, researchers need to complement cognitive knowing with embodied and enacted knowing. They as well need to combine the formal and science-based language of what is seen and measured, with the poetic, polyphonic and metaphorical language of what is unseen but sensed, felt and envisioned. (Dieleman, 2017, p. 170)*

And yet, there is always this terrible thought, looming like a white elephant in the room, haunting mathematicians like David Hilbert:

What if science, what if mathematics, what if art and philosophy might not be enough to handle every situation we throw at it? What if some questions might forever be unsolvable as - like mathematics - the systems we use in our quest for answers themselves are incomplete? What if we never find the answers to the-essential questions? What if all the computers in the world will not suffice to calculate the right path to follow? What if the answer really is 42, like Douglas Adams' supercomputer *Deep Thought* proposed? In the original Star Trek series, there is a very fitting dialogue between Mr. Spock and Captain Kirk about a computer called "Landru" they just had managed to destroy. Landru is essentially a representation of the following philosophical concept:

Landru is a computer designed to be the perfect ruler for its society, therefore it is programmed to be highly moral. However, Landru is incapable of operating in a way that would enable its people to lead a meaningful life. To keep them from harm, it abolished and severely punished all forms of individualism, seeing this as the only logical way to keep order and prevent conflict. A purely logical machine, Landru is oblivious to human subjectivity and creativity as a vessel of understanding, purpose, and meaning:

> *SPOCK: 'Marvelous.'*
>
> *KIRK: 'What?'*
>
> *SPOCK: 'The late Landru, Captain. Marvelous feat of engineering. A computer capable of directing the lives of millions of human beings.'*
>
> *KIRK: 'But only a machine, Mr. Spock. The original Landru programmed it with all his knowledge, but he couldn't give it his wisdom, his compassion, his understanding, his soul, Mr. Spock.'*
>
> *SPOCK: 'Predictably metaphysical. I prefer the concrete, the graspable, the provable.' (Sobelman et al., 1967)*

Don't Panic!

The answer to everything

Don’t panic. Just grab a towel and stick your thumb up to the stars. In *The hitchhiker’s guide to the galaxy*, Douglas Adams tells the tale of a massive supercomputer which, once its construction has finished, far surpasses the intellect and capabilities of its creators. It is so ultimately potent that it already calculated the vectors of all atoms in the universe since the Big Bang in the blink of an eye. It regards itself as the second greatest computer in the whole universe because it "can navigate the infinite delta streams of future probability and see that there must one day come a computer whose merest operational parameters I am not worthy to calculate, but which it will be my fate eventually to design." *(Adams, 2017, p. 169)*

Its name is *Deep Thought*. Baffled by their own creation, the two programmers who invented and built the machine seek out to find a task that is worthy of the most powerful AI in the universe:

> *'O Deep Thought computer,' he said, 'the task we have designed you to perform is this. We want you to tell us …' he paused, 'the Answer!'*
>
> *'The Answer?' said Deep Thought. 'The Answer to what?'*
>
> *'Life!' urged Fook.*
>
> *'The Universe!' said Lunkwill*
>
> *'Everything!' they said in chorus. (Adams, 2017, p. 170)*

This is, of course, what one would want such a computer to do. To dive into the big questions which have plagued mankind since the beginning. What is the meaning of really anything? We became pretty good at explaining the "how?"s the universe throws at us, and our bare knowledge and its distribution have grown exponentially over the past decades *(Batovski, 2008)*. The thing we are still very bad at are the "why?"s. For the "how?"s, we have invented science and reason. But for the "why?"s, we made up a lot of stuff over the years, called some of it religion, some of it philosophy, and there is much in between that does not fall into any clear category.

Given the indeterminable nature of many of our existential "why?" questions, their answers might very well forever remain undecided, or in some other form incomprehensible for the confines of our minds. So why not at least give it our best shot and create a computer that is so immensely powerful that even the computer itself is an iteration of many layers, far surpassing its own base layer of programming intended by its creators. It might be a creation that far exceeds its own creators intellect and thus becomes incomprehensible to them. Why not use such a hypothetical being to feed it the ultimate "why?"s to find the answers to all the big questions we as human beings so desperately want to be answered during our sparse time on this planet?

However, even if the answers to these questions *could* be calculated, much like the number of pi "could" be calculated *(Kidwell, 2020)*, will we ever be able to live and know the result due to their possibly sheer infinite volume? Even if possible, it might very well take an eternity to answer these "eternal" questions due to the potentially endless computational power needed to finish the calculations. We would probably never see the day where we would finally be awarded with a definite result.

Deep Thought answers:

> *'But the program will take me a little while to run.'*
>
> *Fook glanced impatiently at his watch.*
>
> *'How long?' he said.*
>
> *'Seven and a half million years,' said Deep Thought. (Adams, 2017, p. 173)*

Only a few million years

Seven and a half million years is thankfully very little time compared to eternity or the age of the universe itself, which is 13,787 billion years *(Tillman, 2017)*, or roughly 1838 times the timespan Deep Thought needs to calculate in the book.

So seven and a half million years later, the crowd gathers in joyful desperation longing for the final answer. Thousands of generations have since passed on the legacy of the day Deep Thoughts will provide an answer for everything there is. Finally, the moment is here:

> *'Never again,' cried the man, 'never again will we wake up in the morning and think Who am I? What is my purpose in life? Does it really, cosmically speaking, matter if i don't get up and go to work? For today we will finally learn once and for all the plain and simple answer to all these nagging little problems of Life, the Universe and Everything!' (Adams, 2017, pp. 177-178)*

What could a calculated answer to the meaning of everything even look like? It could very well lie beyond what our languages and tools for comprehending and understanding "what is" are capable of. Still, it might be translated into an expression of its implication, could be translated to another, ultimately even more fundamental question, could be translated into art in the broader sense, where understanding is a purely emotional and reflective process without the necessity of any syntax, could be a totally indecipherable mathematical formula following axioms we have not yet discovered, or even something completely different than all of these combined.

> *'Forty-two,' said Deep Thought, with infinite majesty and calm.*
>
> *A slow stupefied silence crept over the men as they stared at the computer and then at each other. (Adams, 2017, pp. 181-182)*

Deep Thought's answer to Everything is 42. This might seem like an elaborate joke of the author at first. And while it indeed is a humorous remark about the very nature of the relation between such questions and their respective answers, it actually is far more than that. It is a funny way to point at a problem very rarely conceived.

42

So let us assume the answer to *everything* is indeed any natural number. Whatever the answer might be (given something like "an answer" or "the question" even exists), we have to start asking ourselves: Do we even ask the right questions? We take it for granted that asking questions in any syntax we see fit will reproduce satisfying results. But what if we are completely on the wrong track? The whole point of Deep Thoughts cryptic answer "42" in the book is that the beings who constructed, programmed, and instructed the most powerful computer in the universe fell short on actually formulating the right questions to compute it in the first place. Instead, they only asked for "an answer to everything", a hypothetical *God's formula,* if you will. They never considered not being able to formulate the right question and the consequences that would render even the right answer to everything completely useless and beyond our comprehension or as Deep Thought put it:

> *'So once you do know what the question actually is, you'll know what the answer means.' (Adams, 2017, p. 183)*

Maybe we need computers built by computers for even formulating the questions we seek to be answered. I want to conclude this chapter with a philosophical thought:

Can we ever truly be sure that even *if* we find the answers to some of the "why?"s, will those not be constrained to our very own little computer inside our heads? Maybe these answers can always only be a trace of what truly is, a shadow in the means of Plato's cave, shrouded in an unliftable veil of darkness that will sustain throughout eternity, forever out of reach for organisms such as ourselves. It might be buried deep inside a construct much more complex than any of our languages will ever be able to describe in any satisfying way. We might have to come to terms that the future of searching for these answers is undecided at best. We might come to terms with the issue that our questions are naive and not even close to what substitutes the "why?"s. Being able to formulate the right questions that should be asked is thereby also intrinsically an undecided matter. As Fernando Sols, a professor in the field of quantum physics at the Complutense University of Madrid states:

> *Science cannot offer a complete explanation of reality because of the existence of fundamental limits to the knowledge that it can provide. Some of these limits are internal in the sense that they refer to concepts that belong to the domain of science but are outside the scope of science. The 20th century has left us with the formulation of two important limitations of scientific knowledge . On one hand, the combination of Poincaré's non-linear dynamics and Heisenberg's uncertainty principle leads us to a picture of the world where reality is, in many ways, indeterminate. On the other hand, Gödel's theorems reveal the existence of mathematical theorems that, while true, cannot be proved. More recently, Chaitin has shown, inspired by the work of Gödel and Turing, that the randomness of a mathematical sequence cannot be proven (it is 'undecidable'). (Sols, 2010)*

So is that it? Are the formal systems we are working with to explain the phenomena of the world like mathematics bound to forever be flawed systems? Do we just have to get used to the idea that some answers might forever be in the dark? Or can we maybe formulate an answer, a satisfying idea that substitutes the need of proving the obvious? Can we articulate the flaws and incompleteness of our own methods into something more useful, maybe even into something that is as real as the answer we might have expected in some corners of the unexplored realms of our minds?

Can we think of what is necessary to make peace with not knowing something while accepting that we actually can dance on the dark side by changing the question and letting the answer be something different? Could there be a tool to formulate an answer to an undecidable problem or system in a way that closes the gap of something inherently incomplete? Could art do anything like that?

Maybe art can, in a way, teach us that the right answer does not always have to be the correct one, but instead can be anything, as long as we can imagine it. It can act as a bridge for closing the gap between knowing and understanding and enables us to journey further ahead than pure facts and knowledge let us assume. In my view, art can serve as the vehicle for traversing and exploring meaning which transcends knowledge and science.

Plato's arcade

An artwork by Sam Ekwurtzel revolved around "things half hidden". It was essentially a pair of video loops, which depicted sets of close up pictures of TV screens *(Genocchio, 2008)*. These were TVs sold over eBay or other private sales channels. Ekwurtzel noticed that many times when people were taking pictures of those TVs in order to sell them, the glossy dark surface of the bulged TV screens would catch all of its surroundings in its reflection. So if you zoomed in on the unsuspecting picture, you would suddenly find yourself immersed in a second layer on top of what the picture was intended to depict. Often, the person who took the picture and also the entire room they took it in could be seen clearly in the reflection of the screen, certainly more often overseen by its creators than not. I see it as a metaphor for reality often reflecting back at us when we least expect it. That is why I also find it very interesting that in the context of the intentionally surreal, we often willingly accept the abstraction of an unknown object because the result of the abstraction is much more interesting then its "real" components. We often find ourselves choosing the illusion rather than reality. In a sense, it is as New York Times art critic Benjamin Genocchio puts it when talking about a one minute long, unnamed video loop which is part of a series called *Between Now and Forever* by William Lawson *(Antenna, n.d.)*. It depicts a surreal stop motion video of a person sliding face down through an urban environment:

> *What I also like about this video is the way in which it plays with our willingness to respond positively toward that which we know isn't real. This is in some ways the opposite of what Plato was talking about, for it involves a knowing appreciation of something clearly artificial as if we are heading back into Plato's cave just for the fun of it. (Genocchio, 2008)*

I wonder if Plato would have agreed with the following statement: Practicing art is as if heading into Plato's cave just for the fun of it. I would even argue that we turned Plato's cave into an arcade of some kind. We have become masters in creating digital illusions to entertain ourselves, especially because they let us experience things that are not real, worlds that we otherwise would not be able to explore. Sometimes I even get the impression that we, as a species, do not even want to really *be* in "the real world". We constantly try to escape it with new technologies like smartphones, video games, virtual and augmented reality, just to name a few, creating a mirage of our real selves and reality *(Suler, 2015)*. It seems that the children who once fled Plato's cave find themselves as refugees, desperately trying to unlearn everything that awaited them outside and returning home to a colorful and representative place where they can set up and melt-in with their alter egos.

John R. Suler is a professor of psychology at the Rider University in New Jersey. He is one of the most prominent researchers in the field of psychology in the context of the digital age and coined the term *cyberpsychology*. In his book, he writes about us humans becoming interconnected with every possible device and appliance:

> *Thanks to our creation of these interconnected electronic devices, we humans developed the ability to manifest our ideas, customs, personal identities, and relationships with others in a space filled with buzzing electrons that we controlled. Humans had become electric. (Suler, 2015, p. 4)*

This is Plato's arcade. We don't want to be ourselves in our purest form, we want to be what we imagine we could be in another impossible and yet undecided life.

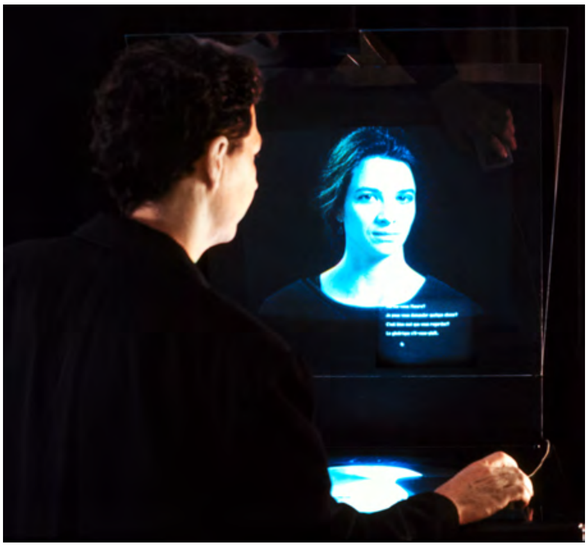
Plato's portrait

Luc Courchesne is a Canadian artist and interactive media designer, widely recognized for his innovative approach to the fusion of technology and art. One of his most famous works is called *Portrait One*, which has been shown at the Montreal Museum of Fine Arts for the exhibition *e-art: New Technologies and Contemporary Art, Ten Years of Accomplishments by the Daniel Langlois Foundation (Foundation Langlois, n.d.)*.

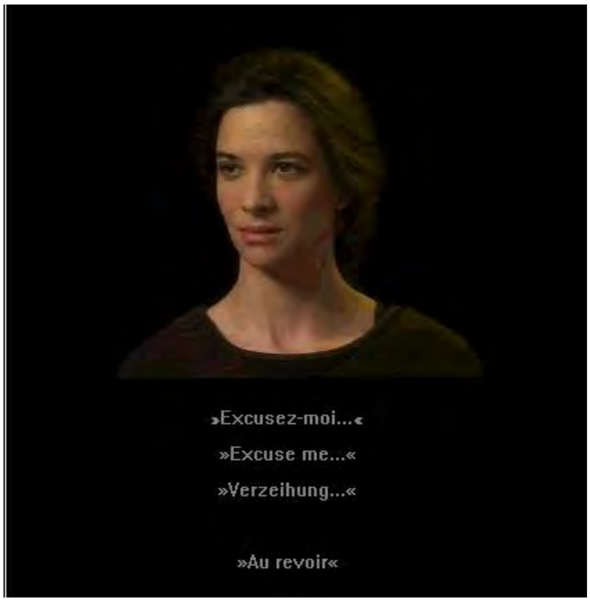
Portrait One is an interactive digital installation that was first exhibited in 1990. It consists of a projection on a large piece of glass that displays a life-sized portrait of a person called Marie. The visitor is standing in front of the screen, and the otherwise inactive portrait, which does not appear to be concerned with its surroundings, only comes to life when the user decides to interact with it. Through the visitor pointing the cursor at possible choices of dialogue, the portrait of Marie begins to recognize the user as an individual inside her space, whom she then offers to get involved in an intimate conversation with her. Although it seems like an evolving pattern for the user, due to the possibility of choosing follow-up questions during the dialogue, this freedom of choice is only an illusion. The complete conversation and its possible variations are predefined by the artist and unfold in a completely predictable way. The answers exist prior to the questions.

Luc Courchesne's artwork was all about creating an illusion. And it more than succeeded, since it fooled one unsuspecting visitor after another and was at the same time also a homage to the novel *The Invention of Morel* by Adolfo Bioy Casares. In his book, Casares tells the story about a machine found on an isolated island that is able to reproduce a lifelike experience for an uninformed viewer which is almost impossible to distinguish from reality itself for the narrator, despite having metaphysical properties *(Mambrol, 2023)*.

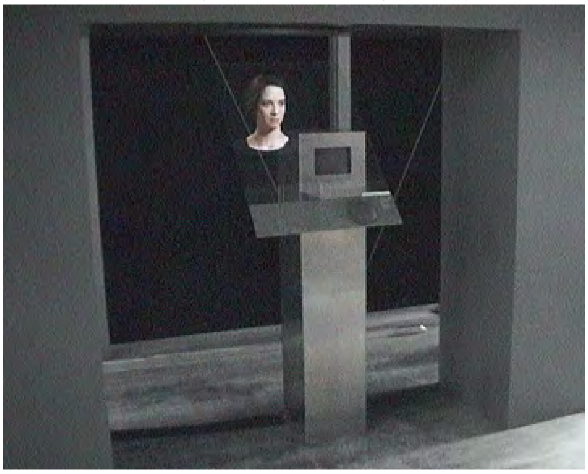
Figure 6, "Portrait One" by Luc Courchesne



(*Portrait One*, 1990)



(*Portrait One*, 1990b)



(*Portrait One*, 1997)

(*Portrait One*, 1990c)

So in my view, the concept of Portrait One can be classified as an illusion of a presumably undecided system, as the user is not aware and cannot be sure of its deterministic way when interacting with it for the first time. It is therefore a digital metaphor for a reflection in Plato's cave that adapts to theoretically unlimited interactive choice. It presents and disguises itself as a system influenced by external factors beyond the artist's control, including an unpredictable outcome. The viewer becomes part of the artwork as an active participant in the creation of the piece itself. Their choices are combined with the subject of the portrait, creating a hybrid installation that is unique to that particular moment. The result is a seemingly constantly changing and evolving work of art that is influenced by the viewer's actions. By creating an installation that is actively influenced by external factors, Courchesne relinquishes control over the piece and allows it to take on a life of its own. This approach reflects a larger shift in contemporary art at that moment in time, where, as Bernar Venet already hinted in a previous chapter, the emphasis is on the process of creation rather than the finished product. In conclusion, Portrait One was an innovative and thought-provoking work of art that exemplifies Luc Courchesne's unique style at that point in time. By making use of a seemingly undecidable system in the form of evolving dialogue, Courchesne creates an illusion and a peek into the feature of computer art where interactive installations are able to constantly evolve and change while including the environment they exist in into their evolving patterns. I think what he wanted to show with this piece in the early 90s is that the use of technology and the active participation of the viewer creates a far more immersive experience than traditional approaches in media art, which challenge exactly these traditional notions of art and the role of the artist in the creation of his work. To sum this up in the context of this chapter, Portrait One can also be philosophically connected to Plato's cave in several ways:

As mentioned before, Plato's cave is an allegory of the way in which people perceive reality, and it suggests that what we see and understand is not necessarily the truth. The idea is that we are like prisoners in a cave, and the shadows or reflections that we see on the wall are all we know of reality *(MasterClass, 2022)*. Within the context of this installation, the viewer's choices of engaging into a dialogue with Marie are superimposed onto the subject of the portrait, thus creating a hybrid experience that blurs the line between reality and representation. This can be seen as a metaphor for the way in which we understand reality itself. Like the prisoners in Plato's cave, we only see shadows and representations of reality, and we are unable to fully grasp the truth. Moreover, Portrait One challenges the idea of a fixed and stable identity by creating the illusion of a constantly changing and evolving image or person. This can also be connected to Plato's idea in that our understanding of the world is constantly shifting and that what we perceive as reality is, in fact, a rather fleeting and unstable construct *(Wilstrup, 2023)*. Furthermore, the interactive nature of Portrait One can be seen as an analogy for the way in which we interact with the world, since it is a much more limited and constrained experience than talking to a real person. It is confined within a fixed framework of space and time. In Plato's cave, the prisoners are unable to interact with the world outside of the cave in a similarly constrained fashion and thus are limited to only interact with the shadows on the wall. But what does that mean?

As stated before, our perception of reality is shaped by our own experiences within our subjective consciousness *(Lu et al., 2020)*. This means that everything we think about the world is merely the imprint of its shadows and can never be a representation of its truest form. The true form of anything must remain an undecided mystery to us, as we are always limited at least by our own perception. We can never know if any form or shape we conceive as a complete concept in reality only is a mere construct of relations between our mind and the natural world. We are the ones defining its form and structure. That is the essence of us giving meaning to our own existence by constantly interpreting the incoming stream of data and manipulating the output by processing it through all the filters we have constructed in our mind and then reiterating every bit of it again and again, creating an infinite loop of self-reference of the information that we process in a circular fashion. And what goes around, comes around.

A circular mystery

Come in and have a pi

In order to further elaborate on the word "undecidable" in the context of this paper, I want to demonstrate its broader implication on a more known topic: The number (or rather mathematical constant) we call pi, which is also directly and indirectly a very prominent building block of classical and digital artworks. It is a distinguished component when it comes to forming the landscapes we navigate, both on canvas and on screen. It has also been the inspiration for many stories and conspiracy theories throughout history *(Danesi, 2020)*.

As explored in the book *Pi (π) in Nature, Art, and Culture - Geometry as a Hermeneutic Science*, it draws its fascination partly from the mysteries that surround its origins while also echoing the success of many universal applications up to this day. Circles, spheres, and cylinders have always been some of the most practical and important geometrical forms due to their impact on culture, invention and architecture. Their geometrical properties have defined how we see reality and our place in the universe itself. Even the atom has been imagined by the ancient Greeks as being some kind of spherical object that can not be further separated (hence the name: átomos/ἄτομος means indivisible). A sphere as the center of the hypothetical building blocks of the universe *(Van Melsen, 1960)*. While some believed (and oddly, some still do) that the earth is flat, they did not imagine it as a rectangle. They imagined it as a round disk because, as I would argue, their observations of the cosmos and its many inhabitants pointed towards being at least circular, if not spherical by nature.

So it comes as no surprise that early astronomers also imagined the cosmos as a collection of spherical and cylindrical entities evolving around themselves and each other in spherical relationships *(Thurston, 1996)*. A circle has no beginning and no end. It became a symbol of the ever reoccurring cycles of biological and physical processes like day and night, the seasons of a year, birth, death, and reincarnation. The list goes on. While the concept of its form probably was a concept generally known throughout the ages (although it was not properly defined by any reproducible geometrical or mathematical understanding), it took a while until mankind did its first steps into understanding the very geometrical properties of it *(Kvasz,1998)*. And from the geometrical perspective one cannot really have a circle without understanding pi.

The secret of pi

So what makes pi so special? To understand why its concept is so important, we have to go back to its very origins. The "discovery" of pi as a constant can be dated as far back as about the year 3000 - 2000 BC:

> *[The] Babylonians and the Egyptians knew more about π than its mere existence. They had also found its approximate value. By about 2000 BC, the Babylonians had arrived at the value π = 3*1/8 and the Egyptians at the value π = 4*(8/9)^2. (Beckmann, 1976, p.12)*

One of the real highlights of this seemingly mundane revelation of the constant is precisely that it is a constant. During the Neolithic period, it was uncommon to measure things using fixed constants. Modern mathematics had not yet been invented, and people did not usually put numbers in relation to each other. However, they did know how to make practical use of proportions:

> *If the volume of a stone is doubled, the weight is doubled; if you run twice as fast, you cover double the distance; if you treble the fields, you treble the crop; if you double the diameter of a circle, you double its circumference. (Beckmann, 1976, p.11)*

The concept of a ratio being inherently constant to the variations of its proportions was not known or recognized to a sufficient degree to these people. This changed when:

> *[S]omewhere along the line some inquisitive and smart individuals must have seen something in common in the behavior of the magnitudes in these and similar statements:*
>
> No matter how the two proportional quantities are varied, their ratio remains constant *(....) And thus, man had discovered a general, not a specific, truth. (Beckmann, 1976, p.11)*

Objective truth, not

So there we have it: the birth of a so-called "general truth". In my view, a general truth is as close as one can get to an objective truth. Pi is certainly an interesting candidate for being an irrational number describing an abstract constant that is considered generally and thus objectively true. It is probably universally accepted and has been applied in formulas used countless times. It has never disappointed. The only major problem is that no one was ever able to verify if it actually is objectively true. Probably, no one will ever be able to do that. Let me summarize the above: Pi is a mathematical constant which is described by an irrational number which we consider a general but unverifiable truth.

So what exactly is pi, if not a verifiable, objective truth? Let us take a step back and look at its plain definition:

> *The number pi is a mathematical constant used to describe the ratio of the circumference of a circle relative to its diameter. It is also classified as an irrational number, which means it cannot be defined by the ratio of two integers. (Encyclopaedia Britannica, 1998)*

This means that pi is always only an approximation of the ratio of two numbers, because, in terms of mathematical accuracy, this is close enough for many cases. And that is where the more attentive reader will recognize the very nature of what pi really is and where I am heading at with this approach. In my view, it must be classified as an intrinsically undecidable system because of its approximate nature. We know how to theoretically calculate indefinite digits of it, but at the same time we will never know what the final result will really be. Even if we started today, even if we had started a thousand years ago with all the hypothetical *Deep Thought* computers we could imagine, we cannot know if or when pi could be completely calculated, how many digits it has, if it is infinite or not, or, in other words, if what we think it is is even verifiable. One might conclude that we have to accept that there might exist seemingly objective truths that are and will never be verifiable. Or do we? As a Reddit poster going by the name "Brian" puts it in a post about the topic:

> *No. We're intrinsically subjective observers of reality. The fact of objective truth existing isn't in conflict with this. Potentially everything that we believe to be true could be wrong - we have an inherently subjective viewpoint, rather than any kind of direct access to objective truth. Even if that were the case though, it wouldn't mean there are no truths, just that we've misidentified which they are. (Brian, 2014)*

So even the idea that something could potentially be objective or "true" cannot be sufficiently verified within our own minds and is thus also an eternal subject in the realms of undecidability.

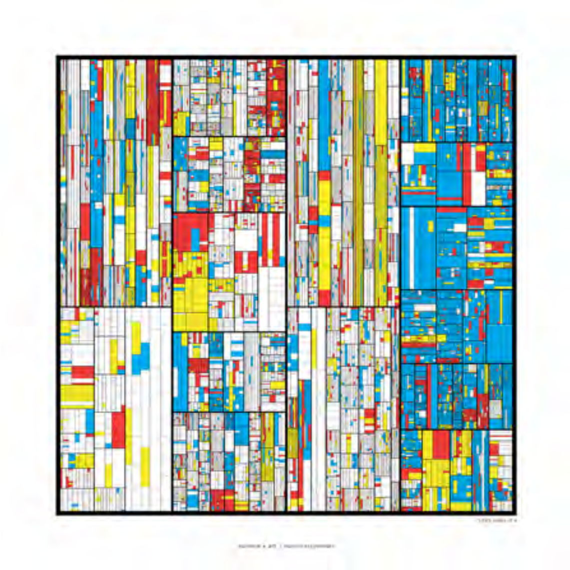
Not another pi chart

There are many interesting artists and artworks who have different approaches to the general fascination with pi, such as Martin Krzywinski. Before becoming an artist, Krzywinski was a scientist, which he still is today. While working as a medical physicist at the Genome Sciences Center in Vancouver, he and his colleagues soon realized his special talent for data visualization. His main focus lies in preparing his research data analysis to efficiently and aesthetically communicate it to a broader audience. The GSC's website states about Krzywinski: "He is an artist. His canvas is a computer and his medium is data." *(BC Cancer Research Institute, n.d.)* That goes in line with what he has to say about himself: "'It is hard to describe my role as a

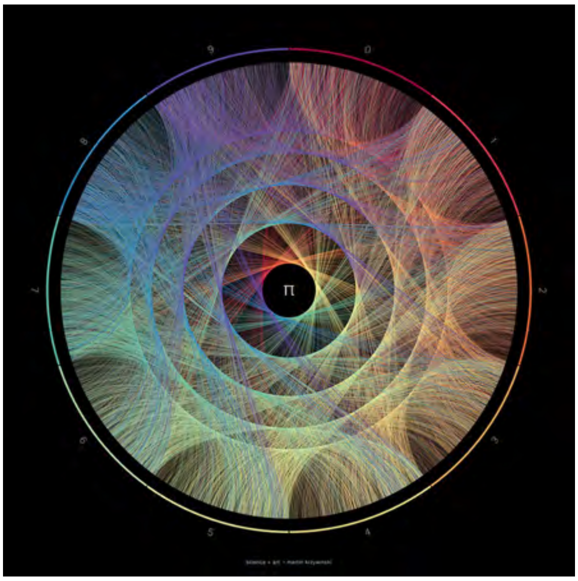
Staff Scientist at the GSC,' says Martin, 'Some days I am a science artist and other days I am an artistic scientist.'" *(BC Cancer Research Institute, n.d.)*

The following pictures are some examples of his many approaches to visualize the sequence of numbers that encapsulate pi as a whole. Even in their abstraction, they stand on their own as artistic data sculptures without having to hint towards their mathematical inheritance from the most famous constant. While I will not go into the exact details of every example in figure 7, I will illustrate Krzywinski's train of thought when creating visualizations like these.

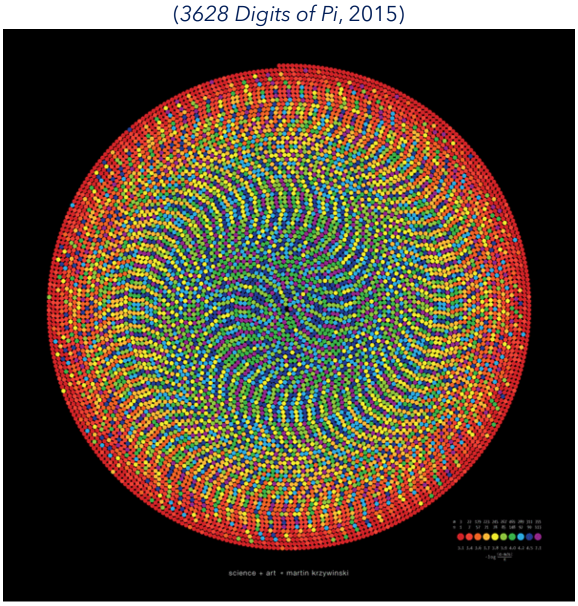
Figure 7, "Approximations of Pi"
by Martin Krzywinski



(*3628 Digits of Pi*, 2015)



(*Pi Transition Paths #2*, 2017)



(*10000 Approximations of Pi*, 2014)

(*Forest of the Digits of Pi Desolate*, 2021)

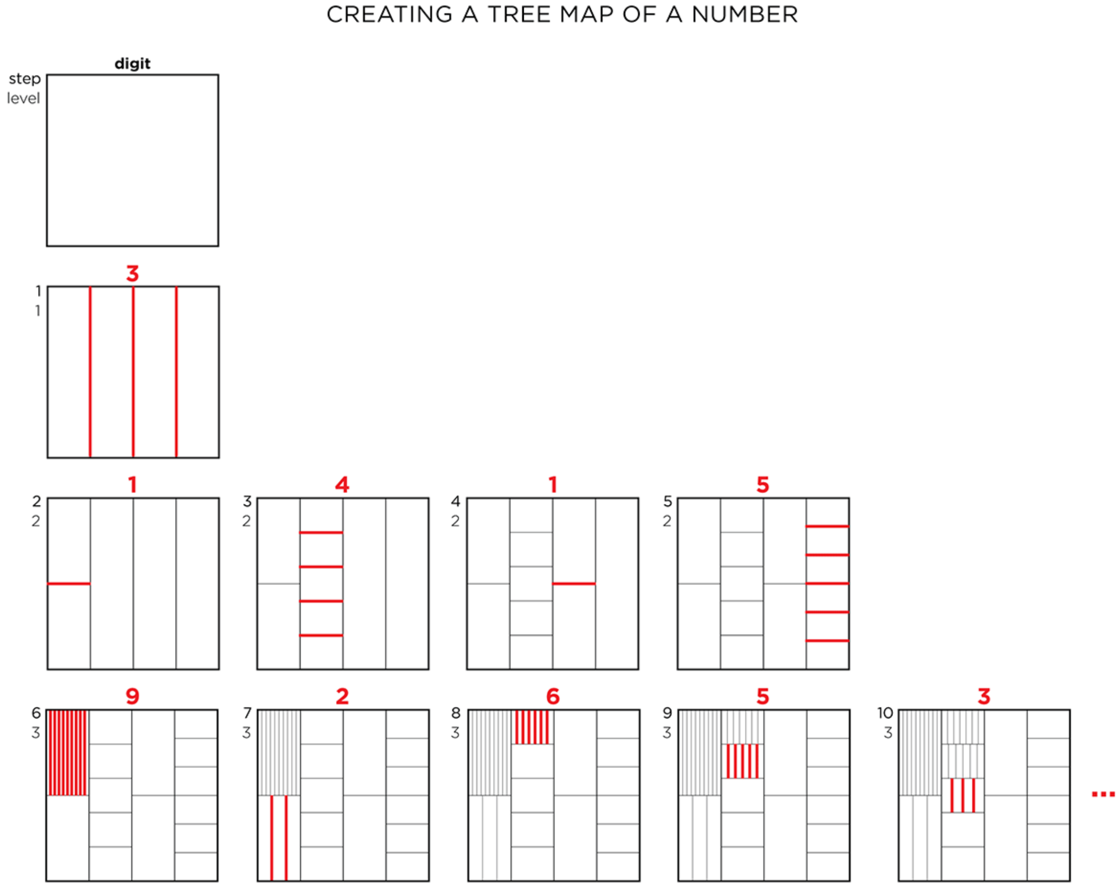
The top right picture shows a mandala-like data visualization. To achieve this result, Krzywinski wrote a program to connect consecutive decimal numbers with a colored line while also referencing a color value. The colored circles on the outside represent the number of total transitions between the successive digits. In the context of pi's very probably infinite count of decimals the amount Krzywinski used is only marginal. But it is still enough to generate some kind of pattern throughout the mapping process. This is indeed a very interesting feature since it seems to appear to manifest in a more regular shape compared to what one would expect from the seemingly unpredictable randomness of each decimal. I would probably attribute this occurrence to the law of "Gaussian distribution" which in this case means that the average of all values are a dominant factor in their visualization *(Weisstein, n. d.-a)*.

In another example, Krzywinski used a method called "treemapping". This method is usually used to visualize information in a hierarchical way, such as the correspondence between file sizes on different storage devices.

> *We begin with a square and progressively divide it. At each stage, the digit in PI is used to determine how many lines are used in the division. The thickness of the lines used for the divisions can be attenuated for higher levels to give the treemap some texture. (BC Cancer Research Institute, 2023)*

See figure 8 on the next page for a better understanding:

Figure 8, "Tree Map of a Number" by Martin Krzywinski



(*Creating a Tree Map of a Number*, 2015)

The interesting thing about this example in my view is the conversion to a geometrical pattern not directly connected to the geometrical origins of its attributes. It is creating something similar to the aesthetics of a binary system with rectangles and squares combined to an organic but at the same time artificial shape. The constant pi, which is normally used primarily in calculating circular geometry, is transformed into a division of squared sectors. It is an artistic transformation of its nature, and the resulting piece also gives the illusion of pi being somehow constrained. It creates an impression of pi as a finite entity that can be squared off, constrained within borders. It gives a sense of deterministic "completeness" and decisiveness. Of course, this is only an illusion, but a beautiful one.

The important takeaway is that, from a mathematical point of view, the process of calculating every single decimal of pi is of course theoretically possible. But the process in its entirety would probably end up being infinitely undecided *(Kidwell, 2020)*.

In fact, there has been an effort going on for some time by many different institutions using different technologies to give it a serious try to go as far as possible in the discovery of pi. As of 2022, Google has actually set a new benchmark in this endeavor *(Iwao, 2022)*. They were able to calculate pi to an accuracy of its 100th trillion decimal which is about three times as much as the former world record. The process took 157 days, used about 515 terabyte of storage and processed roughly 82,000 terabyte of data. Yet one might wonder, what is the point of these endeavors anyway. Because we now know that we may never find what we hope for; like completing the endless sequence of decimals within pi. According to Google, it was basically only a very elaborate proof of concept:

> *As a developer advocate at Google Cloud, part of my job is to create demos and run experiments that show the cool things developers can do with our platform; one of those things, you guessed it, is using a program to calculate digits of pi. (Iwao, 2022)*

Most real world applications of implementing the constant rarely exceed the need for more than 10 decimals. Even NASA has no need for more than 15 decimals for its most advanced projects like calculating interplanetary navigation data *(NASA/JPL Edu., 2022)*.

Still, calculating its seemingly endless digits of decimals, combined with its undecidable nature and the general concepts of infinity lead to an interesting thought experiment which I want to iterate on in the next chapter. This concept I will introduce would also apply to every infinite and non-repeating decimal number or sequence.

Infinite ideas

The DNA of everything

Pi is an infinite, nonrepeating decimal - meaning that every possible number combination exists somewhere in pi. Converted into ASCII text, somewhere in that infinite string of digits is the name of every person you will ever love, the date, time, and manner of your death, and the answers to all the great questions of the universe. Converted into a bitmap, somewhere in that infinite string of digits is a pixel-perfect representation of the first thing you saw on this earth, the last thing you will see before your life leaves you, and all the moments, momentous and mundane, that will occur between those two points.

All information that has ever existed or will ever exist, the DNA of every being in the universe, EVERYTHING: all contained in the ratio of a circumference and a diameter. (kenfoldsfive, 2012)

In 2012, a user on the forum platform Reddit asked for the "most mind-blowing sentence you can think of", which someone by the nickname "kenfoldsfive" answered with the above quote. What started as a semi-philosophical answer on Reddit soon became a popular discussion subject around the internet. Let us dissect his answer and find out what the essence of this mind-boggling statement actually is. It has little to do with pi itself, because as stated before, the same statement could be true for every other irrational number which consists of infinite non-repeating decimals. Other than that, pi is perfectly suitable to talk about the broader implications of the statement, serving as a representative of any such number. Let us consider this:

If pi really *is* an infinite irrational number with non-repeating decimals (which means that there will never be a point in its calculation where it gets stuck in a sequence of decimals repeating itself), then it would produce any possible combination of numbers as an arbitrary sequence that there could ever be. Which means that one could find every single number or sequence of numbers in it at some point in time, past, present, or future. This means that it must also contain all the numbers one ever came in touch throughout one's personal life, like one's birthday, bank account, lotto combination, etc. In fact, it means that literally everything that can be described by our arithmetic system in numbers throughout the universe must, at some point, be contained somewhere in this vast ocean of decimal sequences. Of course, it would also require an infinite amount of time if one were bound to search for and find the information they seek, as such is the very nature of infinity.

This is why the kenfoldsfive's answer also entertains the claim that, if pi were "[c]onverted into a bitmap", it would be a pixel-perfect multimedia representation of *literally* everything. Moreover, this would not only be true for indefinite "real" variations of ourselves and our lives and universes; "real" meaning a hypothetically distortion-free, logical and cohesive stream of "data" without the slightest alteration of our impressions of reality: Like watching a perfect digital copy in a resolution matching the receptors in our eyes at a 1:1 receptor:pixel ratio, without any artifacts and in full 3D.

This also means that when we look at the random nature of the single parts that make up these sequences reoccurring throughout infinity, the underlying undecidable structure of this system itself must carry within it the "DNA" of abstracting the very reality in which it exists. It is an abstract concept of itself. A short analogy to understand this better: We fully understand the concept of a circle. It is an equally curved line whose ends are connected to each other. This would also be true for an artistic approach, if we talk about a circle as a baseline for an artistic element, i. e. drawing a picture. Its concept acts as an identifier which can be individually recognized even in very abstract visual artworks. But as soon as we try to look at it from a scientific point of view, it eludes every attempt of successfully applying our available tools in order to fully describe it *(Beckmann, 1976)*. It is, in itself, an abstraction of reality. This happens in the exact moment we try to describe it with mathematics or geometry. Within this abstract layer of its undecidable constants, we must assume that there must be yet another layer of abstraction, a glitch in the system if you will, since the circle itself is described by an infinite irrational number with non-repeating decimals. So every sequence which would describe any arbitrary part of reality at the same time also exists in every single one of its slightest alterations of itself within the endless stream of data as a direct result of its undecided nature.

It's glitchy

This also means that every possible so-called "glitch" we could ever perceive within such a hypothetical construct of our own existence is also encoded within such decimal sequences. It means that any artistic attempt at abstracting this representation of our realities is already determined to be encoded in this infinite data set as well.

We can conclude that anything that exists within one of these data streams can lie in between a broad spectrum of perfect, 1:1 representations of everything which is indistinguishable from reality itself, and a level of ultimate distortion where nothing is left beyond an absolute abstraction of a moment in time. Both of these versions can of course intertwine from frame to frame, or stay perfectly consistent throughout infinity. To clarify further, imagine a snapshot of exactly one single moment which is saved in your mind and experienced as your reality in that precise moment. This snapshot would be, in all its volume and "data" your mind has collected in order to give context to it, also be found encoded, if at all being translatable to values represented by numbers, in an irrational number with non-repeating decimals like pi. Of course, this is again only true if pi actually *is* a sequence of non-repeating decimals, which also remains undecided.

Moreover, every conceivable glitch, like, for example, surreally distorted versions of the audiovisual aspects of that moment's "snapshot" would simultaneously also always be represented somewhere within such a number. Imagine a complete and utter distortion of what you have, are, and will ever experience to any arbitrary degree. This is, of course, also valid for every other conscious entity similar to us in the cosmos. Every possible version of reality of every possible parallel universe would be encoded as a representation of itself in an infinite and non-repeating stream of data. *(Hürter & Rauner, 2011)*.

This process of further abstracting an already "in-place" abstraction of reality can also be an intentional method of reflecting on meaning and motive in an artistic approach or on a meta level. For example, one approach in the field of art concerning the distortion of data is summarized under the term "glitch art". To understand this term, we have to dissect it into two parts and find out what a glitch actually is:

> *A glitch is a short-lived fault in a system, such as a transient fault that corrects itself, making it difficult to troubleshoot. The term is particularly common in the computing and electronics industries, in circuit bending, as well as among players of video games. More generally, all types of systems including human organizations and nature experience glitches. (Wikipedia contributors, 2024c)*

At this point, it is worth noting that there is a considerable difference between something being a glitch in a system or being a bug. To clarify this briefly, this is what Alex Pieschel wrote about the matter in an article of *Arcade Review*:

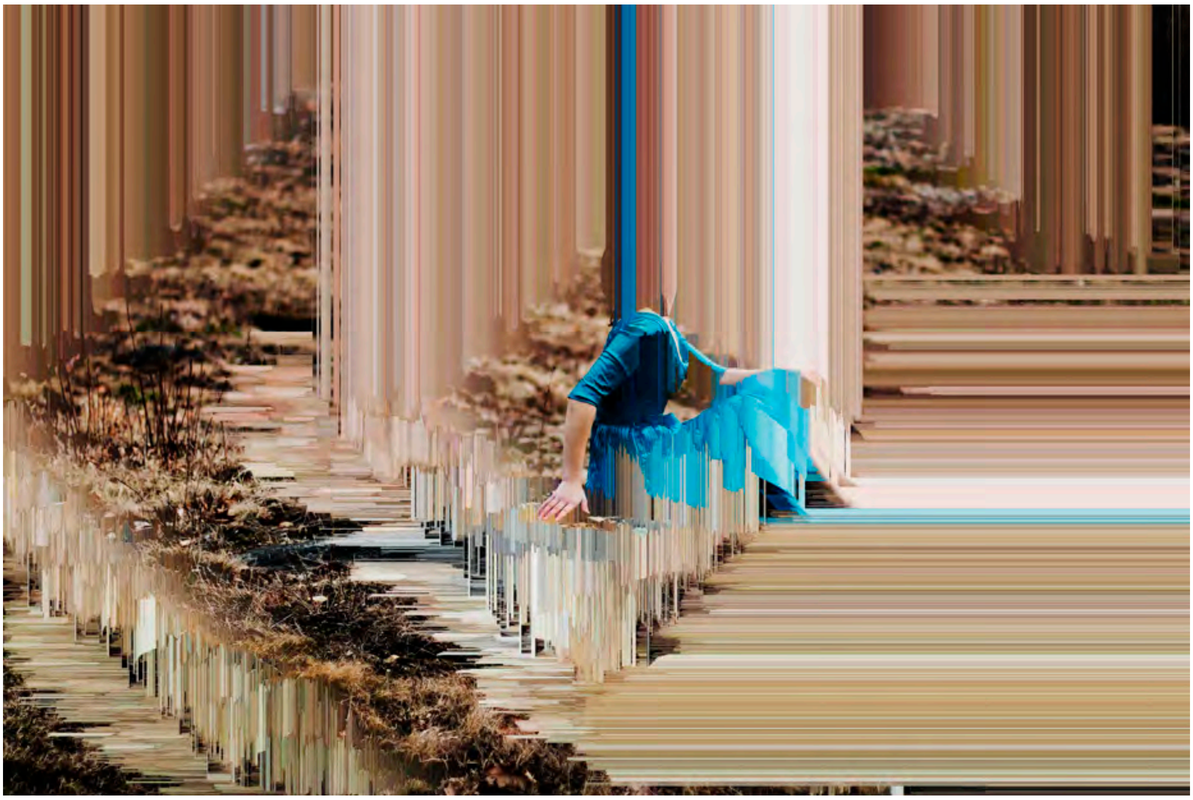
> *In general internet nomenclature, both words refer to errors that work against authorial intent, but "bug" is often cast as the weightier and more blameworthy pejorative, while "glitch" suggests something more mysterious and unknowable inflicted by surprise inputs or stuff outside the realm of code. (Pieschel, 2014)*

It is basically mostly, but not exclusively, an audiovisual disturbance or distortion, at least in the field of multimedia experiences. It is a short or long term error that manifests itself and is witnessed by an observer. In other terms, glitch art can be defined as a "… visual style that is distinctive in its use of artistically curated digital distortions." *(Marinho, 2022).* Or, as Ben Barnhart from *Vectornator* puts it:

> *Normally, these brief interruptions to our films or video games are annoying interruptions that quickly solve themselves and disappear from the screen. Glitch Art is the exploration of what it looks like when those glitches don't self-correct. (Barnhart, 2024)*

Figure 9: Aestethics of "Glitch Art"

(*Michael Betancourt - Cinegraphic*, 2013)



(*Sabato Visconti - Glitch6*, 2014)

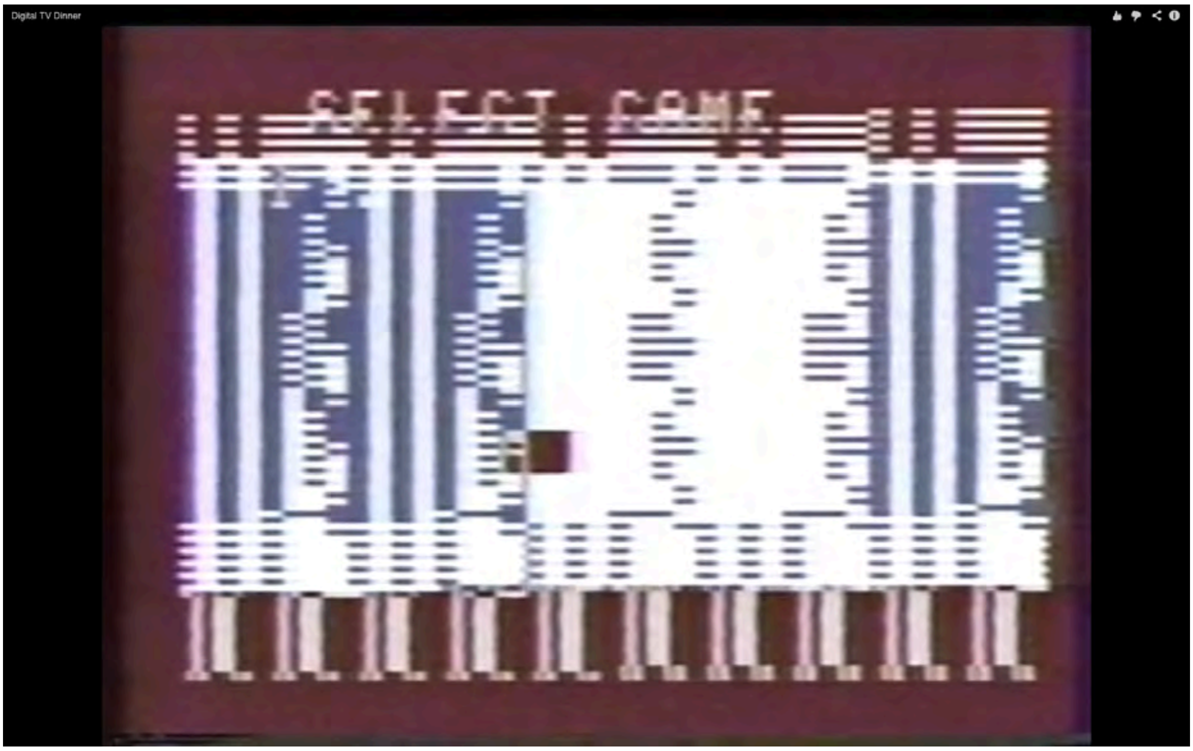
Glitch art has become its own subgenre within digital media art, and although the origin of the term itself is probably a direct result of its occurrence in quantity, quality, and recognition as "glitched art" throughout its history, it can actually be dated back as far as as the late 1960s and early 1970s *(Flux, 2023)*. In 1976, Jamie Fenton and Raul Zaritsky created an art piece that is considered to be one of the earliest examples of the concept in the broader context of multimedia. The piece is called *Digital TV Dinner*. It was created by manipulating a video home console named *The Bally Astrocade (Betancourt, 2017)*, which was not a very popular choice for a video home console, even back in those days.

This is how the archive of *Neural*, a critical digital culture and media arts magazine, explains it:

> *Digital TV Dinner is a video art clip from 1979 created by Raul Zaritsky, Jamie Fenton, and Dick Ainsworth using the Bally Astrocade console game to generate unusual patterns. The Bally Astrocade was unique among cartridge games in that it was designed to allow users to change game cartridges with power-on. When pressing the reset button, it was possible to remove the cartridge from the system and induce various memory dump pattern sequences. Digital TV Dinner is a collection of these curious states of silicon epilepsy set to music composed and generated upon this same platform. (Neural, 2013)*

The result is what could be called a very early adaption of a thoroughly applied spectacle of glitches to an unsuspecting machine. Although it is of course best experienced by watching the videos about it, on the next page are some pictures to just give a quick impression of its aesthetic:

Figure 10, "Digital TV Dinner" by Jamie Fenton and Raul Zaritsky



(*Digital TV Dinner*, 2023)

(*Digital TV Dinner*, 2023b)

While a glitch per se is actually defined as something unexpected or something that is not anticipated inside its respective system, glitch art, on the other hand, implies an intentional act of inducing or exploiting a known fault inside a system to produce or "summon" a glitch as the anticipated outcome or state of the program. This is an important distinction in terminology, because glitch art is not only about exploiting previously unknown software or hardware faults, but it is also a sculptural effort of deliberately manipulating digital material or hardware to get a better glimpse of a particular glitchy moment in time. This means that often also the implied glitch in media art is actually artificially made after the creation of the "output", which means that sometimes it is not even a "real" glitch that produces the audiovisual effect. The glitch is implied in the outcome but not actually present in the source itself. To clarify, one could, for example, take a picture with a camera and manipulate it so that it looks like there had been some kind of glitch in the camera's system while taking it; although there was never an actual glitch in the original picture and the hardware that produced it.

That brings us to the differentiation of possible methods identified within the field of glitch art, as described on glitchology.com, a website dedicated to archive and explain the terminology in broader detail *(Glitchology, 2022)*:

- Databending: the process of manipulating a media file of a certain format, using software designed to edit files of another format. Distortions in the medium typically occur as a result.

- Pixel Sorting: the process of isolating a horizontal or vertical line of pixels in an image and sorting their positions based on criteria such as luminosity, hue, or saturation.

- Datamoshing: a technique of damaging video clips to create a glitch effect wherein frames that should change don't. It's most noticeable between cuts and across motion.

- Software Glitches

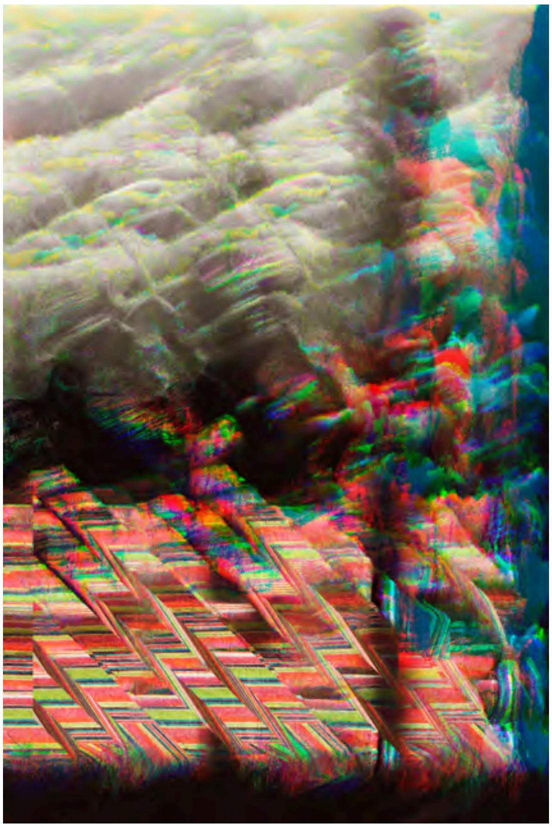
- Hardware Glitches

- Combining Glitches

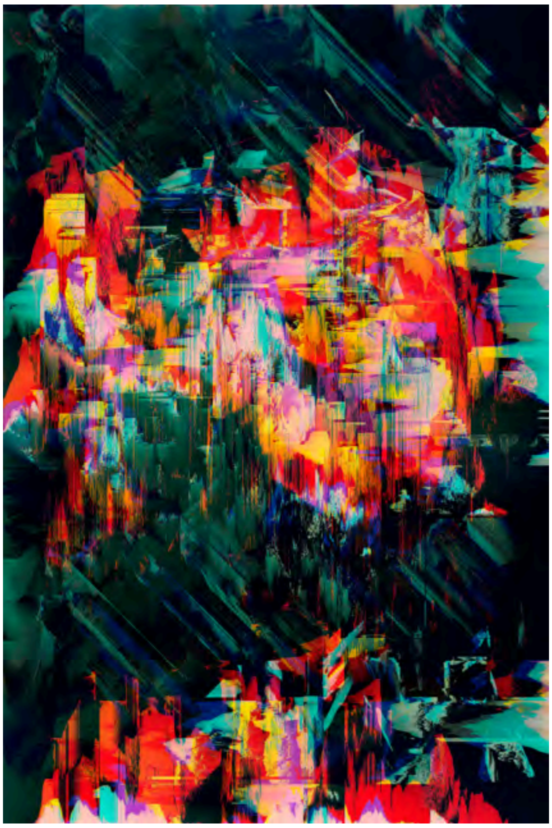
So one could make the argument that using one of the first three methods (i. e. databending, pixel sorting, and data moshing) to superimpose glitches and produce digital glitch art can be seen as more of a homage to the phenomenon compared to the more traditional methods of exploiting unintended software, hardware or combined glitches alike. It is a bit like using using a "paint" filter over a photograph, making it look like it was painted although it actually was not. You see, how can something that intentionally resembles the result of a glitch in the system be the product of that exact glitch if it wasn't the result of a glitch in the first place. Thus, the term glitch art serves as an umbrella term for an aesthetic category in media art rather than defining the constraints of a specific method. Of course, this is only a technical observation on my part, rather than a qualitative one. "Fake it till you make it" is not always used as a shortcut, thus "glitch art" can rather be seen as its own valid genre in which the effort often tops the "make it" part, as far as it concerns finding and exploiting real glitches in the system. Glitch art is about experimentation, not following any hard rules, and that is what makes it interesting.

In a way it can also be seen as some sort of deliberate digital deconstructivism, as it often occurs as the deliberate fracturing and reordering of originally unglitched input; thereby uncovering or creating more interesting properties which were formerly hidden from the eyes of the observer. Of course, with the rise of modern computers, glitch art became ever more popular and began to first take over the underground art and music scenes, while in recent years it has long reached far into the mainstream *(Gitchology, 2022).* Some of the more prominent artists in the field include Sabato Visconti, Michael Betancourt, Rosa Menkman, Nam June Paik, Nick Briz, Ben-Baker Smith and the JODI art collective, just to name a few. In figures 11 - 14, I want to show you some of their respective artworks achieved by applying the methods mentioned above onto digital media on the next page:

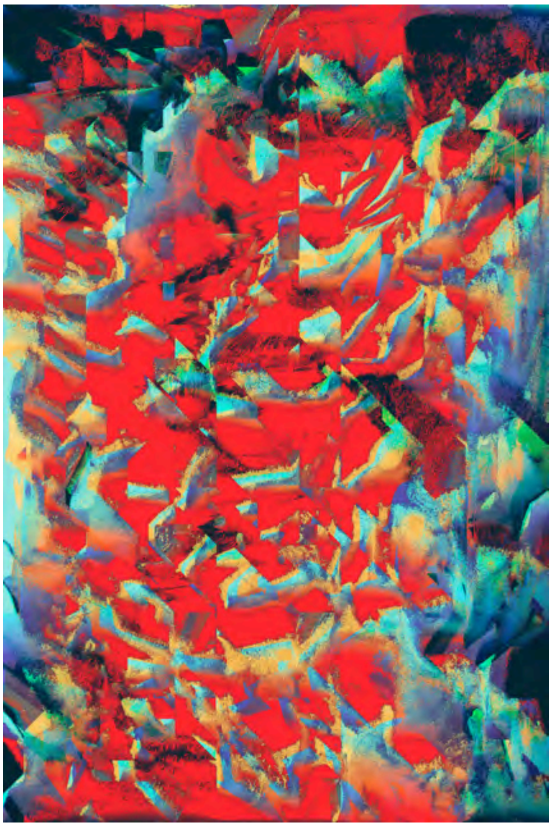
Figure 11, "Images Adrift"
by Sabato Visconti



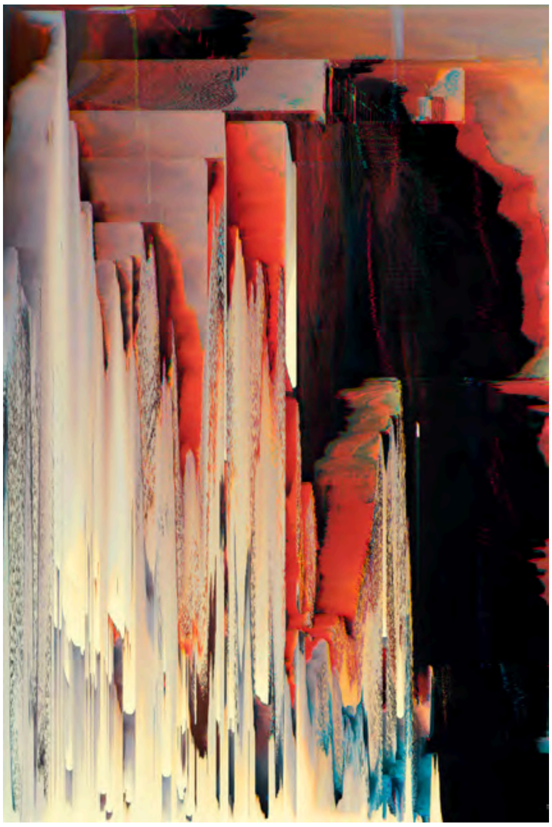
(*Images Adrift*, 2014)



(*Images Adrift*, 2014b)



(*Images Adrift*, 2014c)

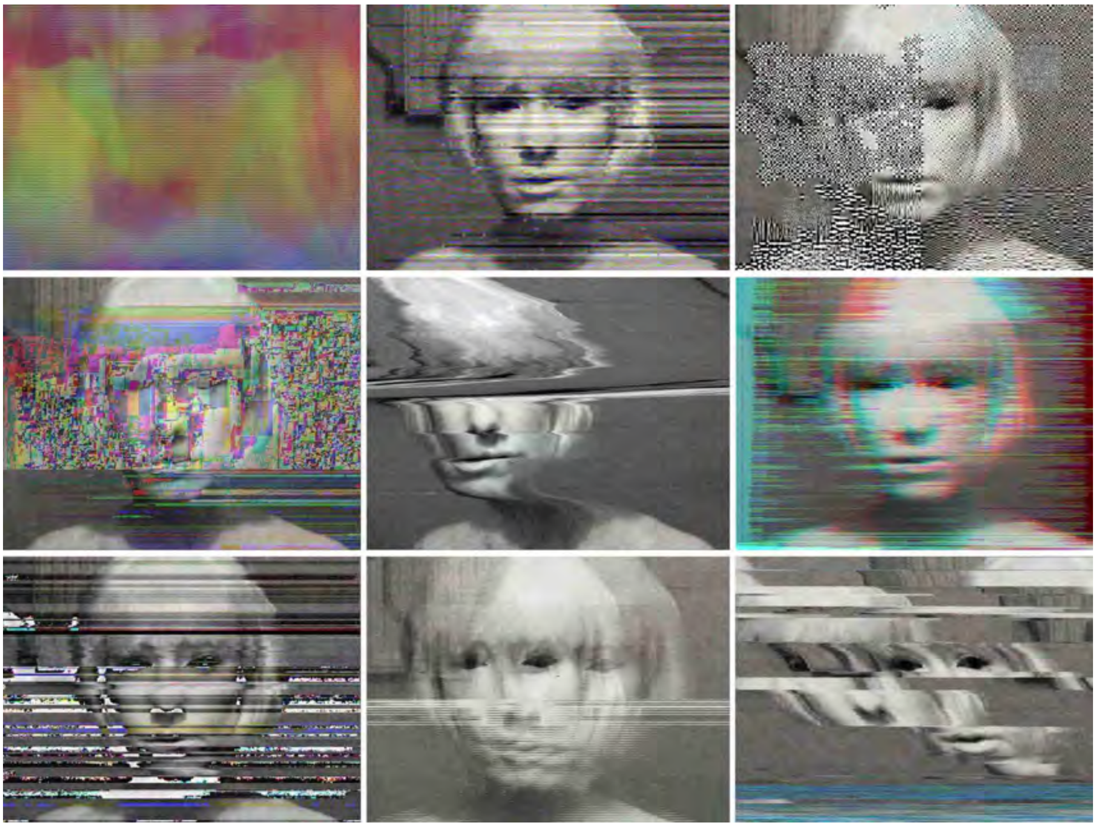


(*Images Adrift*, 2014d)

Figure 12, Glitch Art by Michael Betancourt

(*The Kodak Moment*, 2013)

Figure 13, Glitch Art by Rosa Menkman



(*A Vernacular of File Formats*, 2021)

Figure 14, Glitch Art by Nam June Paik

(*Time by Dance*, 2019)

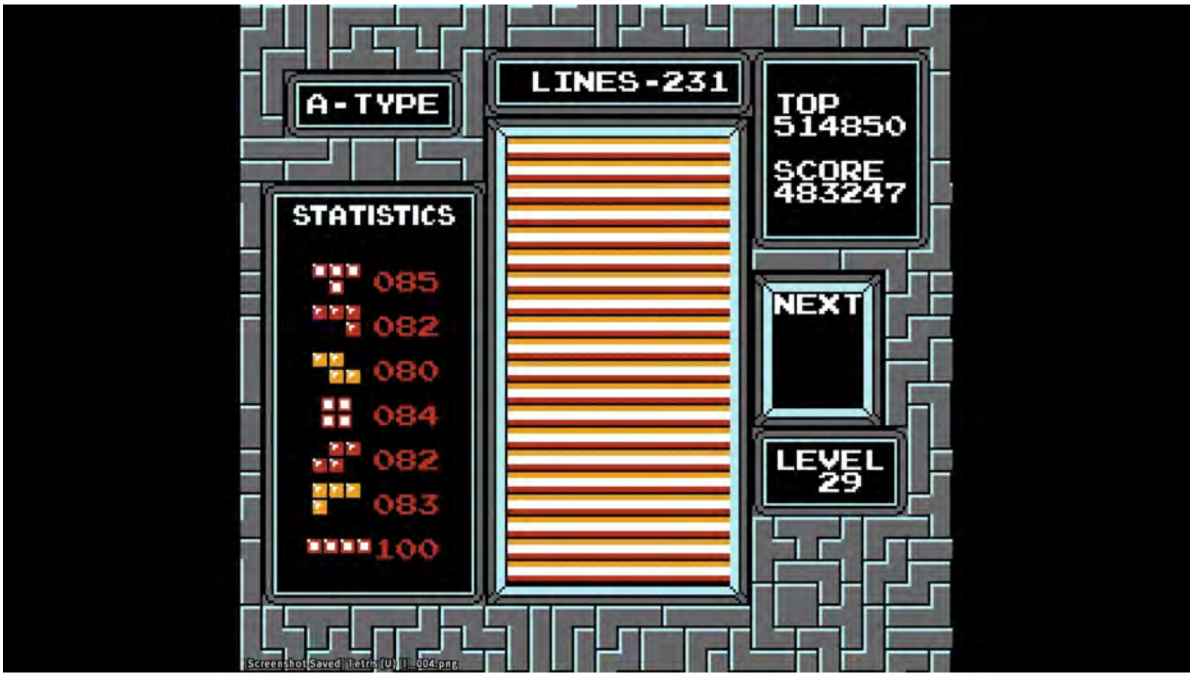
The blocks that mean the world

We all know *Tetris*, a video game invented by Soviet programmer Alexey Pajitnov in 1985 *(Wikipedia contributors, 2024a)* and later made famous by Nintendo in the dawning area of home consoles. Some love it, some hate it, and many never even played it; but it has become part of the collective, global memory. So much so that it is considered by many to be one, if not *the* definitive video game of all time *(Brown, 2023)*. Much like chess, it is simple to learn but very hard to master. It was first released to the public in 1989 and up until this day, people are playing it, often with more compassion than ever before. But has anyone ever beaten it? The answer seems obvious at first, because many would consider Tetris an unbeatable or open-ended game, given its "high score" nature which was a classic and predominant feature of many arcade games of its time. It seems like a perfect analogy for the everlasting trope of "man vs. machine", raising the question if humans could ever reach or even surpass the impossible feats of algorithmic machine logic. It almost seems as if nobody ever even expected to "beat" *Tetris* in the first place, only striving to get the highest score and earn bragging rights over one's gaming companions. But things have changed, as the following example shows:

Tetris was beaten on December 21, 2023 by the 13-year-old Willis Gibson a.k.a. "Blue Scuti" *(Deb, 2024)*. He achieved this feat by being the first person to reach level 157 in the game. There is only one problem: There is no definite final level in *Tetris*. The game is programmed to be completely open-ended. Players start at level 0 and with every cleared ten lines, they advance to the next level of the game in a fluid transition. With every level, the game becomes progressively harder as the speed with which the blocks falls down the screen gets higher and higher. The fastest speed is reached at level 29 and although it is still possible to advance in levels, the speed stays the same *(aGameScout, 2023)*. So is Blue Scuti just an incredible player, then? Well, its not that simple.

Tetris on the Nintendo Entertainment System is played with a controller in hand. On this controller, there are buttons for controlling the left, right, up, and down movements of the blocks. At the maximum speed in level 29, holding down the button is not enough to move any of the blocks quickly enough to a position where it would be possible to clear any line anymore, thus leading to players always losing the game automatically once they have reached level 29. So for decades, this was considered the end of the game, since it was humanly impossible to advance further and level 29 became known as the "kill screen" of *Tetris* *(Codex Gamicus, n.d.)*. Although it was not a kill screen in a traditional sense. A kill screen in retro games is defined as "when it is impossible to get any further due to a glitch in the game's code" *(aGameScout, 2023).* There is no glitch in level 29 of *Tetris*. It is theoretically possible to clear this level if one could somehow figure out a way to move the individual pieces faster than the game's internal speed allows when holding down a button.

Figure 15, Tetris Level 29 "Kill Screen"



(*Tetris Kill Screen*, 2020)

In 2011, more than two decades after its initial release, a man named Thor Aackerlund became the first person to develop a technique on the exact same hardware to finally achieve a breakthrough and reach level 30. *Hypertapping* involves vibrating one's finger on the buttons of the controller at an incredibly fast pace, as to surpass the game's internal speed *(Patrick Scott Patterson, 2013)*. I really want to emphasize how extremely difficult this is to master this technique: While coordinating the finger movements on the controller to consistently *hypertap*, one also needs to synchronize the position and rotation of the blocks in order for them go exactly where they belong in a matter of split seconds. What seems even more fascinating is that people who knew about the "unbeatable" stage 29 of the game still did not want to admit defeat to the machine and have tried tirelessly for 22 years to find a glitch in the system that would help them to finally achieve what was thought impossible in the first place.

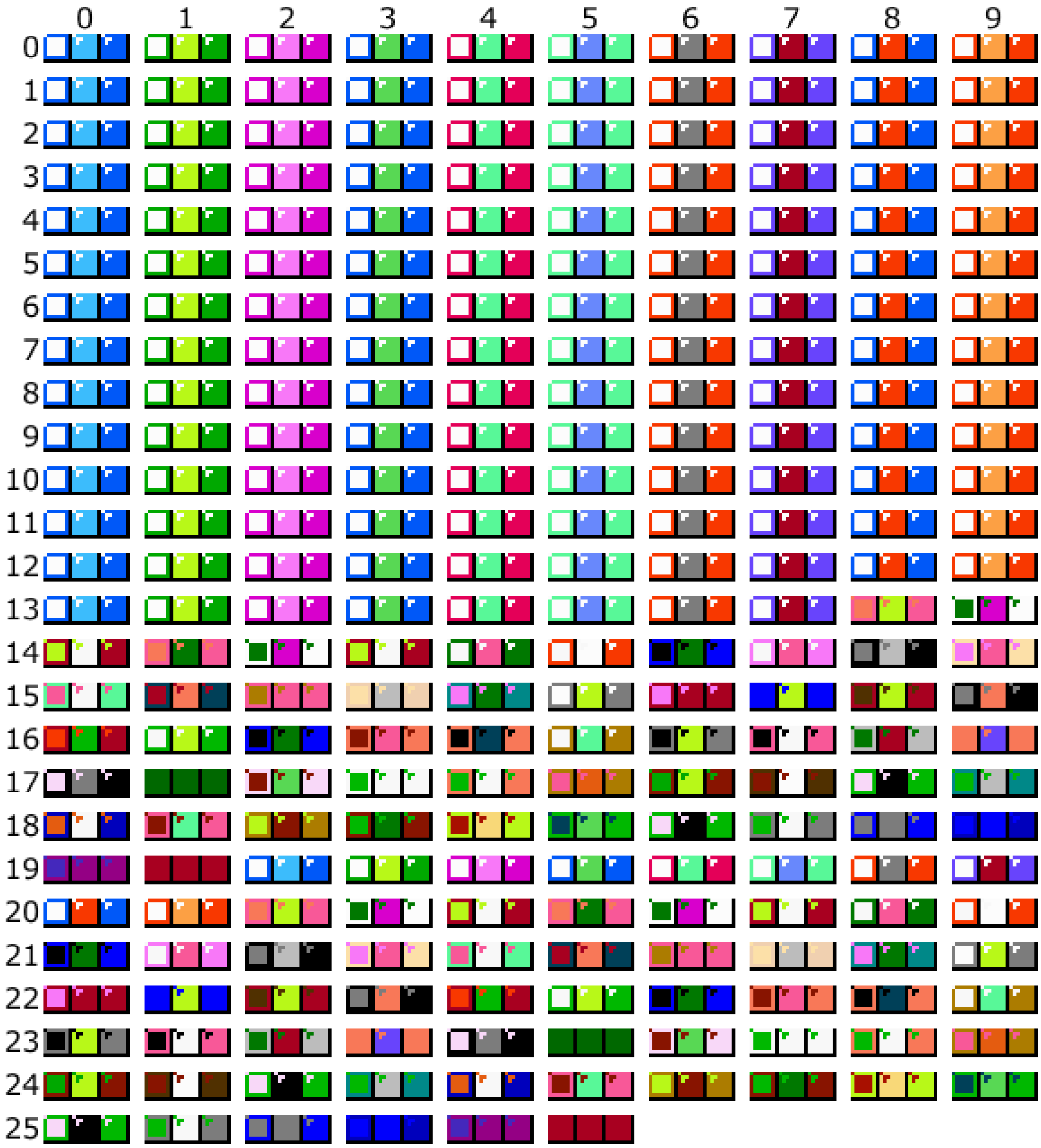
So once this hurdle was taken and knowing that the speed of the game would never go any faster past that level, the sky seemed the limit concerning breaking high score after high score. And for some time, this is exactly what happened. But just to illustrate the difficulty of finishing level 29 and reaching level 30, Aackerlunds world record stood for over seven years, he himself was never able to advance further. On September 24, 2018, a player named Joseph Saelee was the first to break through to level 31. Over the following years, he slowly improved further, up to level 36, before being overtaken by an even faster player, nicknamed "EriclCX", who even made it to level 38 *(aGameScout, 2023)*. Yet there was a problem: While effort and skill were the prerequisite tools for the *hypertapping* method that was needed to advance through the levels, it was still extremely difficult to do so, which made everything above level 29 a test of endurance and luck. It was in no way an effective method for making any significant and reliable progress, since it was still simply too ineffective to play that way in the long run.

So in 2020, another player nicknamed "Cheez" developed a new method of manipulating the controller buttons which he called *rolling (Wingfryer, 2024)*. This new method consists of "rolling up" one's fingers underneath the edge of the controller, pressing the whole controller up into the other hand and thumb, thus activating one button with the passive hand while simultaneously vibrating one's thumb to activate the other buttons with the main hand. Yes, it is as confusing at it sounds. Nevertheless, this technique is twice as fast as *hypertapping* and requires the player to wear a glove to avoid wear and tear on the skin and extremities. Unsurprisingly, this method is also incredibly difficult to learn, but once mastered, many of the top players in the scene quickly adapted and were able to continuously achieve new records over the next few months *(aGameScout, 2023)*. Now that the game's highest speed had been mastered, anything seemed possible. Reaching any arbitrary high score was only a matter of time and the amount of effort someone was inclined to put into reaching that goal. The race for beating the game and - if there was any - reaching the end, had begun. However, nobody was prepared for the game's next, unexpected challenge: The colors of doom.

> *NES Tetris was originally designed with 10 normal looking color palettes for level 0 - 9 that were bright, colorful and easily visible. Every time you go up 10 levels the game then loops back around to the first color scheme. But because this game was coded in the 1980's, everything was written to try and be extremely memory efficient. So the line of code that is supposed to determine the level color glitches out starting at level 138 and starts pulling data from outside the color table resulting in a bunch of random bizarre color palettes that can be difficult to see. (aGameScout, 2023)*

See figure 16 on the next page for an illustration of these colors:

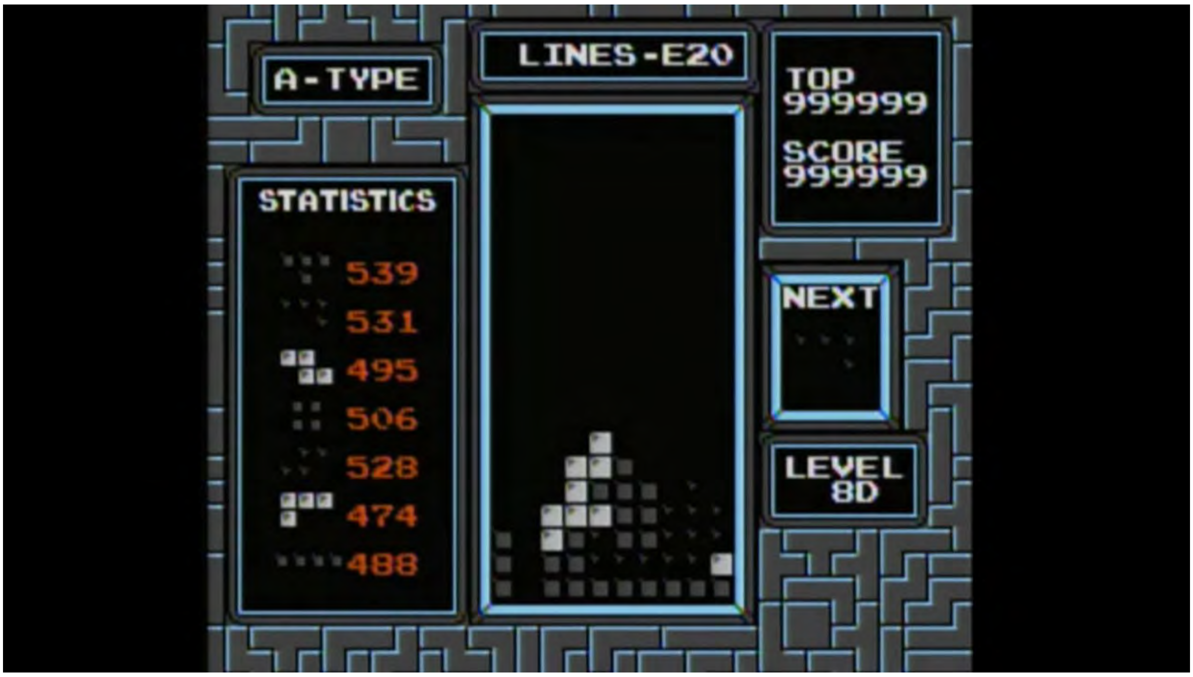
Figure 16, Tetris Original and Glitched Color Schemes



(*Tetris Original and Glitched Color Schemes*, 2014)

At level 146, these unintentionally bizarre color combinations make it extremely difficult to see what is actually happening on the screen because the colors become so dim that the different game pieces become very hard to identify *(aGameScout, 2023)*. While it was not a kill screen in the traditional sense, this time it was an actual glitch in the game's programming that led to a kind of bottleneck and exponentially increased the difficulty. Not only did the players have to master the physical methods of the intricate controller manipulation technique, study the game's rules until they knew them by heart and then combine these two skills to complete the game's objectives of putting the right pieces into the right places at incredible speeds while also having the cognitive and physical endurance to proceed this way until level 138. Now, they also had to do all of the above while readjusting all their senses to glitched-out color schemes that would severely impede all of these traits. This was a crucial obstacle for the community; it took over a year for someone to finally beat the first dim level. But celebrations were short, since almost immediately after that one the game produced a level with nearly pitch black colors which was nicknamed the "Charcoal Level".

Figure 17, Tetris "Charcoal Level"



(*Tetris "Charcoal Level"*, 2024)

Something had to be done. The machine must not win. So in 2021, the programmer Greg Cannon made a *Tetris*-playing AI called *StackRabbit* *(Davis, 2023)*. Since this AI was not limited by physical or mental constraints, it was able to pass through all of *Tetris*' levels with ease. After *StackRabbit* had passed most of the bizarre color-glitched levels, far beyond any previous human ventures, *Tetris* produced another, never seen before hurdle: At level 235, it suddenly takes 800 lines to clear a level instead of the usual ten lines *(aGameScout, 2023)*. However, this did not pose a problem for the AI. Eventually, the game started to internally break apart. At level 237, *Tetris* stops operating, resulting in a true kill screen with an overall score of 102252920 points *(Greg Cannon, 2021)*. The unbeatable game had been beaten, if only by another machine. What happened?

Much like the robot from *Star Trek* introduced in the first chapter, the code of *Tetris* starts to get increasingly inefficient with increasing progress. At some point, a glitch occurs, which prompts the game to switch from reading its own code to reading its own RAM as if this were its code. The output of these glitched operations can then result in a *stop* command which immediately halts the game and freezes it in its current state *(HydrantDude, 2022)*. From that point, nothing further will happen until the system is completely reset. At this point, it gets particularly interesting.

The game crash does not always happen. At least not at the same spot or level of the game. The kill screen-inducing glitch can be triggered by very specific events with varying probabilities depending on different factors on the respective level. It seemed as if the machine did not go down without at least a fight, a final stance to undermine the efforts of human players replicating what the *StackRabbit* AI was able to achieve. So in 2022, a massive effort was put into researching all the different possible situations and their probabilities resulting in the final glitch. One of the main researchers is known in the community as "HydrantDude". He created a spreadsheet containing all the discovered possible situations that could result in crashing the game which is updated all the time and publicly available *(Crash Theory, n.d.)*.

Now that the competitive *Tetris* community had the tools for a realistic chance at ultimately beating the game, it was time for the final battle of man vs. machine. In a frenzy of attempts, many of the community's best players raced for the first glitch they knew would occur at a probability of 100 %, which was to clear a single line of blocks at the end of level 154. On December 21, 2023, player *Blue Scuti* streamed his final attempt at beating the game via the platform twitch, when he reached level 154 *(Classic Tetris, 2023)*. The whole *Tetris* world was watching, as this might become a milestone in gaming history. It would be the first time in more than 30 years for anyone to beat one of the most famous games of all time. *Blue Scuti's* place in the eternal hall of fame of video game history would be certain. And when the moment to put down the last single line clearance needed in order to "win" the game by producing a kill screen finally came - he missed. While panicking to manage all the blocks that appeared on screen, *Blue Scuti* accidentally cleared three lines instead of one, resulting in the game progressing to the next level. Keeping his spirits high, he managed to survive and fight through, aiming to reach the next possible glitch that might occur at level 157. However, this endeavor came with increased risk: The glitch at level 157 only happens with a probability of 73,33 %. When it seemed like all was lost due to misplacing a few blocks, it happened: *Blue Scuti* managed to clear a single line, thereby triggering the first true kill screen that had ever been inflicted on *Tetris* by a human being. He had officially beaten *Tetris*, the unbeatable game *(Classic Tetris, 2023)*.

In my view, *Tetris* theoretically could be described as an undecidable system itself. Although a physical limitation in the form of a maximum speed that would be unmanageable for any human player was intentionally put in place by its programmers to stop players from advancing further than level 29, it was not programmed to "end" at that point. The developers did not think it necessary to implement an actual final level to their game because they considered it virtually unbeatable. So, they left the code incremental and to its own devices once level 29 was reached, unaware of what would happen if somehow, someone would actually "beat" their intended "final" level. All the glitches that happen afterwards, including the probability of generating a halt command by pure chance due to the game's interchanging of RAM with internal code, were not intended by its creators. For decades, they were also not known to any player. *Tetris* was just assumed to be unbeatable, although theoretically endless in terms of game design. It was actually a real example of the so-called "halting problem" which will be introduced in more detail in the chapter "A halting problem" (p. 116). For decades, up until the point where an AI was created to "beat" Tetris, the question if the program would go on for ever or halt at some point after level 29 was undecided. Even now, as far as the research on "true" kill screen inducing crashes goes, it is obvious that some of these crashes only occur with a certain probability in a specific scenario.

Arguably, *Tetris* is a game that was long considered unbeatable, only to ultimately be beaten by its own, undecided properties. While a theoretical *Turing Machine* has no limitations in its hardware's abilities, for its real-world counterpart *Tetris* it was precisely such restrictions that ultimately decided its own, specific halting problem. If *Tetris*' internal programming was not confined to a physical cartridge or console system, similar to the original Turing machine, the outcome of an infinite session of *Tetris* would still remain undecided. I will explain more on why that is later on.

Infinite apes +1

Glitches aside, I want to return to the idea that in any given system with undecidable attributes like the previously discussed, non-repeating decimal pi, almost anything is possible. As long as some kind of randomness and an indefinite timeline is involved, one can get any arbitrary result within that given system and its confines.

To illustrate what this means, I will briefly introduce another example which is set up with just three ingredients: a randomizer, infinity, and letters and symbols of the alphabet. The following thought experiment is based on a theory called the "Infinite Monkey Theorem" and is further elaborated in an article by mathematician Maike Elisa *(Elisa, 2021)*:

Imagine an ape sitting in front of a typewriter, randomly pushing the keys. He has no conception of the alphabet or language, he just pushes any keys he wants. Most of the time the resulting sequence of letters on the paper will not be anything that makes any sense in a linguistic way. But if you just let him do it for long enough, there will eventually come a moment where something in this random sequence can be identified as a word or even a sentence. Just by pure chance, much like winning the lottery. Now imagine infinitely many apes hammering away for an infinite amount of time on infinitely many typewriters. What would happen is that eventually, by pure chance, one of the apes would accidentally write the whole *The Lord of the Rings* trilogy, the combined works of Shakespeare or any other conceivable book. This happens because everything that can happen inside its own logical system, actually *will* happen at some point, given an infinite timeframe. This is true for every event inside the system, even for the ones which have an almost infinitely low probability. You only have to play lotto long enough with the same set of numbers to eventually win. The only problem: It could take you a few lifetimes more than you actually have.

The critical point to this phenomenon is that there has to be some kind or variation of infinity attached to the system. Thus, we can conclude that the statement concerning everything in the universe being encoded within any irrational number with a non-repeating sequence of decimals must be true if, and only if, it is truly infinite. And here we, as proficient adepts of undecidability, can proudly postulate: The answer to the question if pi or any other representative of its family actually *is* infinite, is, and probably forever will be, undecided. And how infinite is infinity anyway?

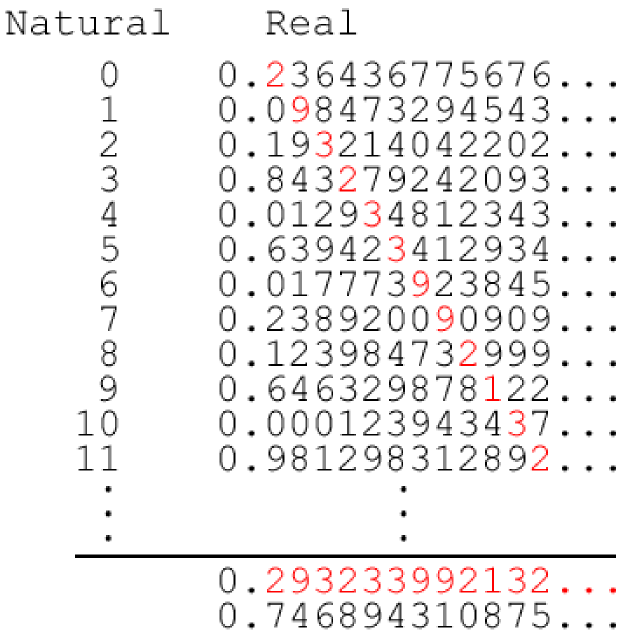
Even if we could prove that there is such a thing as an infinite representative number, we would find ourselves confronted with another mathematical party crasher. The pure concept of something being infinite is already very hard to grasp. Even though the human brain is not capable of computing any sufficient concept to even imagine it to any reproducible and agreed upon degree *(Tatera, 2015)*, there is proof that there can be multiple infinities which have different dimensions *(Chodnicki, 2021)*. And I hope the universe is sincerely sorry for that. So there is no "definitive" infinity as an absolute mathematical or artistic concept. Out of two infinities, one can be longer (bigger?) than the other. How can this happen?

One could be expected to respond, "What nonsense! Infinite is infinite and infinite + 1 is still only infinite." "Wrong!", it whispers from the grave of Georg Cantor, a brilliant mathematician of the 20th century. He actually proved that there are indeed infinities of different dimensions *(Cantor, 1892)*. While his proof is a rather abstract mathematical approach, it is actually not nat difficult to understand at all, even for someone like me who is not particularly good at algebra.

I will therefore try to summarize this idea as simply as possible. As mentioned earlier when explaining Russell's paradox, we can think of things in sets. A set contains things defined by a common denominator for each specific set. For this example, we can imagine a set that contains all the natural numbers. Because we could theoretically count up forever, the set containing these natural numbers must be infinite. Now we imagine a second set which contains all the real numbers between 0 and 1. This set must also be infinite as there are infinite decimals in each of its numbers.

Now, we can combine those two infinite sets by writing down all the natural numbers vertically on the left. Then we start to write down any arbitrary real number vertical on the right of the natural numbers. What we get is an infinite list of numbers where each real number to the right corresponds to a natural number on the left. The natural numbers on the left can be considered as an index for the real numbers on the right.

Figure 18, Sketch of Cantor's diagonalization argument



(*Cantor's Diagonalization Argument*, 2023)

So far, nothing has happened. But what we can do now is to create a new number by taking the first decimal of the first real number and adding +1 to it, as demonstrated by the red digits in figure 18. For example, 0.1234 would become 0.2234. Now take the real number below at index 2 and add +1 to the second decimal. So 0.4321 would become 0.4421. We can continue this process throughout the infinite list of our two sets. What we end up with is a number which is always different from every single other number in our infinite set by at least one digit. At the same time, it is also not (and will never be) included in the complete set of real numbers. This kind of scientific proof is called the d*iagonalization proof (MathPages, n.d.)*. We now have an infinite set of real numbers, every single one defined by an index of infinite natural numbers, plus one number that is never included in this infinite set. We have created infinity + 1.

For our case, this means that Cantor has proven that the infinite set of real numbers in this example must be different in size than the infinite set of natural numbers. And to make matters even more complicated, there are *still* more distinguished levels of infinities relative to each other than just the 2 in this example *(Easwaran et al., 2023)*. But as for the scope of this paper, we will stick with the previous example.

So how can something infinite be different in size than something else that is also infinite? This really seems counterintuitive at first, and it might even raise the question if it is not simply a mistake. It is valid to be suspicious about a method that lead to such a bold statement in the first place. What if all these arithmetic roundabouts are nothing but a flawed system that should not interfere with concepts like infinity from the start? It almost seems like Cantor made use of a logical trick to fool the unsuspecting mind into accepting his argument. One could even argue mathematics itself being nothing more than a construct and therefore an invention of the the human mind, consequently not being "real" *(Hayes, 2012);* thus also things like Cantor's infinities are only constructs without an a priori identity. A mere illusion, created within the syntax of mathematics which is used to abstract the question in the first place.

For debating the undecidability of things, it is sufficient to have a philosophical concept of infinity. Any infinity will suffice. It is, in fact, an inherent attribute of undecidable things, because something can not be undecided if the very attributes which are undecided are subject to a finite process of cause and effect. To illustrate: If the ape from the previous example would only be allowed typing sessions of one minute, there would always be a definite result for each sample size of one minute of typing - the outcome would always be decided. The "magic" only happens in two specific scenarios: Either if infinite apes typed for one minute, or if one or more apes typed for an infinite amount of time.

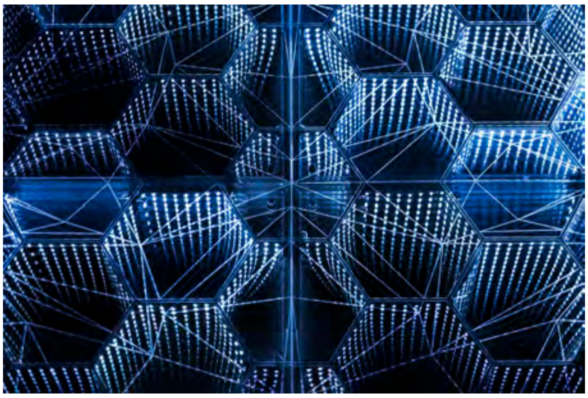
One artist who focuses strongly on the concept of infinity is Gabriel Pulecio. He is based in New York and his works encompass mostly kinetic light installations where he tries to capture moments of infinity and take another visual approach to reality. One of his works is titled *Infinite Tiles of Virtual Space*:

> *Developed as a part of an ongoing research on light and space, Infinite Tiles of Virtual Space is an interactive immersive installation that transforms a space into a vast infinite expansion using light and sound. (Pulecio, n.d.)*

While this art installation is basically a more sophisticated approach to the translucent double mirror principle, it also comes with its own twists and turns. The illusion of infinite space which is created by this method is then also augmented with different interactive and audiovisual elements to transform it into an immersive landscape for exploring. What he is trying to achieve is *"… expanding the user to an immersive environment of virtual three dimensional space." (Pulecio, n.d.)*

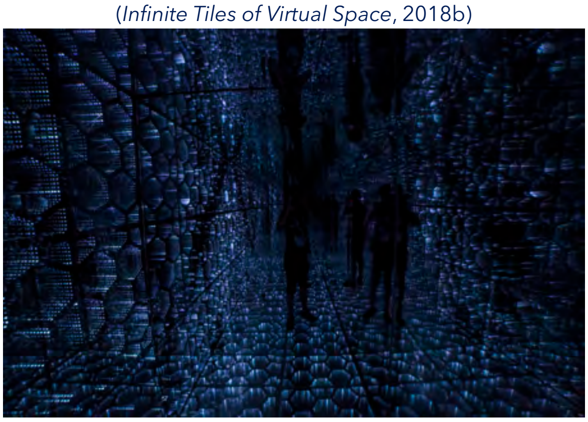
See figure 19 on the next page for some examples of his work:

Figure 19, Artworks on infinity
by Gabriel Pulecio



(*Infinite Tiles of Virtual Space*, 2018)

(*Infinite Tiles of Virtual Space*, 2018b)



(*Infinite Tiles of Virtual Space*, 2018c)

(*Infinity Portal & Six-Thousand Infinity Room*, 2018)

(*Infinite Tiles of Virtual Space*, 2023)

While I personally really like the overall aesthetics of his works, using an underlying concept that is inherently fascinating, like a double mirror, might seem like an artistic shortcut. A professor at my university once said that the easiest way of making "good" photos is to use motives from nature, like animals and plants, because these motives are already aesthetically pleasing. It is much harder to take a good and satisfying picture of something that, at first glance, does not appear that interesting just by itself. The same could be said of the double mirror method. It has an inherent visual effect that already is impressive on its own. Still, I think this example is a very valid approach to visualize and abstract the idea of what conceiving something as being infinite can look like. Its efficacy lies in its simplicity.

As mentioned before, it is very hard if not almost impossible for us to really grasp the concept of infinity. But by interacting and entering into a dialogue with certain artworks or thought experiments, which are contextualizing concepts of infinity, we can at least glance into the abyss and stay still for a moment. It enables our brains to have a brief moment of meditation. Having the time to reflect upon it and project onto it an image that seems to mimic infinity just well enough to be credible for just that moment. It is an example of how infinity could look like inside Plato's cave. And sometimes that is all we want from a piece of art: To give us a reflective moment in order to think about reality itself while believing the illusions that surround us are real.

Journey to Mandelbrot

What at first sounds like some special treat one might take home from an Austrian Christmas market is actually the name of one of the 20th century's mathematical geniuses. Benoit Mandelbrot was a French-American mathematician who is best known for his work on fractal geometry, which revolutionized our understanding of the complexity of natural phenomena throughout many different disciplines. Mandelbrot was born in Poland in 1924 and grew up in France, where he studied mathematics at the École Polytechnique and later at the University of Paris. In 1958, he moved to the United States to study at the California Institute of Technology where he received a masters degree in aeronautics. Mandelbrot earned his PhD in mathematics after his return from the United States at the University of Paris. Besides his stints at Princeton, Harvard, Yale, and the MIT, Mandelbrot worked at the IBM Thomas J. Watson Research Center for 35 years, the majority of his career.

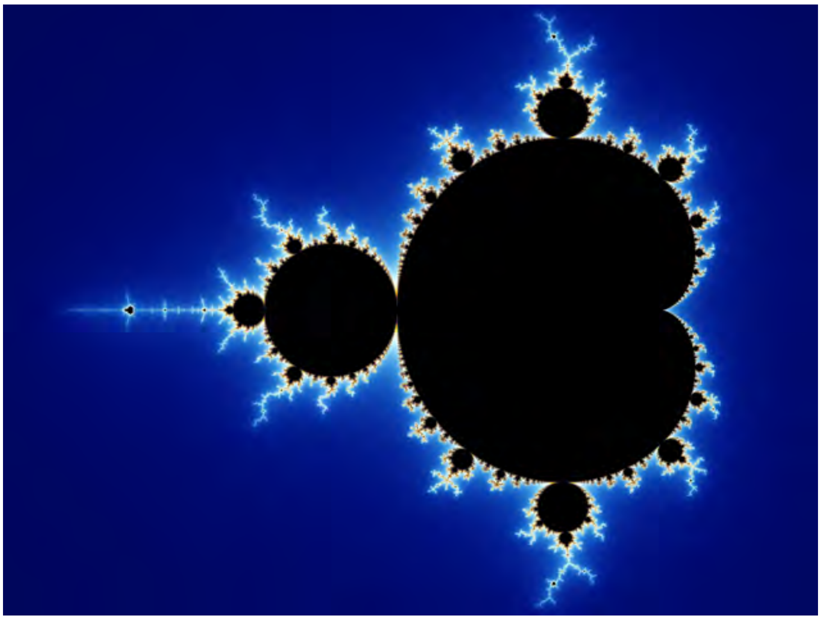
His job at IBM also opened up possibilities to work with these otherwise hardly accessible computers which where still a rarity at that time. Thus he was one of the first to also make use of computer graphics to create and display geometric and fractal representations of his mathematical ideas. This is also when he stumbled upon his now famous invention. Mandelbrot's most significant contribution to mathematics was his development of an abstract fractal geometry called the *Mandelbrot set (Mandelbrot, 2014)*. For better understanding, two definitions are crucial:

- Fractals: Fractals are complex, self-similar shapes that repeat themselves at different scales. They can be found in many natural phenomena, such as coastlines, clouds, and mountains, and have applications in fields ranging from physics to finance *(Fractal Foundation, n.d.)*.

- *The Mandelbrot set*: It is defined as a set of complex numbers that forms a fractal when plotted on a complex plane. The set is defined by an iterative equation that generates a sequence of complex numbers. If the sequence remains bounded, the number is in the set, and if it diverges, the number is not in the set. The points in the resulting set are considered mathematically connected and not computable *(Weisstein, n.d.-d)*.

Figure 20 shows how the Mandelbrot equation f(z) = z^2 + c looks like when visualized as points on a grid, which in this case is a complex plane:

Figure 20, Visualization of the classic Mandelbrot equation



(*Mandelbrot Set*, 2013)

The interesting thing about this pattern and its respective visualizations is that it is somehow branded into our brains and seems familiar enough to most people that they would consider having seen it before. While that is certainly the case, considering its relative popularity in visual programming and computer arts, it also belongs to the broader family of the above mentioned fractals. These kinds of patterns, although very abstract in the case of the *Mandelbrot set*, connect to a more familiar visual imprint of nature itself. This is due to fractals being a kind of mathematical structure seemingly encoded into nature itself *(Zarudnyi, 2023)*. This concept gets clearer as soon as we start looking at patterns in the natural world more closely. Many structures found in the flora and fauna of earth demonstrate recursive patterns, which are especially prominent when observing the way plants grow. Understanding what to look for enables us to reveal natures underlying structure and get access to a level of detail often hidden in plain sight:

> *Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line. (Mandelbrot, 1982, p.1)*

This means that whenever we try to understand the forms that depict nature to use them in our artistic approaches of representing and expressing our ideas, there is a conceptual barrier that at first seems hard to overcome. Like children in a drawing lesson, taking a break from drawing trees with simple geometric shapes on their papers and looking out the class window, we, too, must realize and accept that the concepts of simple geometric forms are often not enough to describe the true form of something. We must forget all the circles, squares, lines, rectangles, and so on, and venture into the realms of chaos if we want to learn the truth. And the truth is that reality often presents itself as something non-linear and unpredictable rather than in orderly forms and sorted patterns *(Frame & Mandelbrot, 2002)*.

"There is no harmony in the universe", Werner Herzog once said, being overwhelmed by the vast overgrowth of the jungle and its seemingly endless complexity of systems and the chaotic struggle for survival of all that lives in it *(Arlind Boshnjaku, 2014)*. And he is not far off from what Mandelbrot's message about fractal relations in nature was about:

> *One of the main messages of fractal geometry is the inappropriateness of this euclidian approach. Nature is understood by its roughness, viewed through self-similarity. (Frame & Mandelbrot, 2002, p. 150)*

Mandelbrot intended to inverse the traditional relation of geometry and nature, which is counterintuitive to the untrained eye, since we like to think about objects as a representation of a large set of complete geometrical forms. These forms can be described as "Euclidian caricatures" *(Frame & Mandelbrot, 2002)*, because they often are sufficient enough to mirror the concept of what we are trying to depict but at the same time will never suffice to accurately represent the true concept of an object. This is because the basic structures of nature itself can be described as something that is infinitely complex in terms of mathematical predictability. So when nature is following this fractal behavior of internal structure and derives its patterns of growth and reproduction out of it, it means that the concept of finite mathematical and geometrical attributes must always be inherently wrong when describing these phenomena.

> *Fractals are infinitely complex patterns that are self-similar across different scales. They are created by repeating a simple process over and over in an ongoing feedback loop. Driven by recursion, fractals are images of dynamic systems – the pictures of Chaos. Geometrically, they exist in between our familiar dimensions. (Fractal Foundation, n.d.)*

The fractal patterns which can be found in abstract complex systems such as the *Mandelbrot set,* as well as the ones found in nature, are inherently recursive and repeat themselves infinitely, which makes their true form infinitely uncomputable. This means that the true form of nature, when it comes to its geometrical representation in space, is more often than not undecidable. This is especially true when we consider that we are never able to know all the initial conditions that drive a complex system in sufficient detail in order to predict its ultimate fate in the first place.

While this is certainly true for the real world, we as humans, crafty as we are, have long figured out that there was something to be found in the patterns exposed to us, even when looking at it without the instruments of modern science and mathematics. While the beauty of something often lies in the eye of the beholder, more often it is exactly these fractal shapes and patterns in nature that we recognize on a subconscious level and it is also these arising structures and forms that artists throughout the centuries were fascinated with. So it comes as no surprise that many of our fellow colleagues in the fields of art have studied and applied these hidden dimensions to their depictions and expressions, even long before we used computers to actually simulate these fractal worlds on a basic level.

> *It's not a secret that objects with fractal attributes are perceived by the human eye as the highest manifestation of harmony and beauty. The crowns of trees and mountain ranges, unique patterns of snowflakes and the "golden" spirals of sea shells and waves, crystals and corals – we are ready to contemplate them endlessly in the wildlife and on the artists' canvases. (Arthive, 2018)*

One of the most famous examples of using fractal elements in a painting is *Kanagawa-oki Nami Ura,* also known as the *Great Wave,* by Katsushika Hokusai, a Japanese artist of the Edo period *(Wikipedia contributors, 2023)*. It also strikes me as a very interesting example because while certainly intended, the fractal elements of the picture do not reveal themselves immediately. They do not seem to be the most prominent feature when first looking at the picture but emerge as the most interesting feature of the whole composition once you explore it further. It is with strange certainty that we consider them beautiful without even needing to know about their fractal secret.

Figure 21, "Great Wave" by Katsushika Hokusai

(*The Great Wave off Kanagawa*, 2015)

The fractal universe in art and culture is vast, so it is futile to reference every single example or every artist who ventured along that path. But I will still introduce a selected few examples which I consider representative for the sheer volume and influence the idea of fractals had, and continue to have, on modern art and culture. While fractal depictions of nature can often be found as a very prominent motive in ancient Asian culture, there are also more modern and abstract approaches to the topic in general around the globe. In figure 22, I want to give two more examples of the intricacy with which ancient painters in ancient China like Wang Meng and Chen Zhou detailed their landscapes:

Figure 22, Landscapes
by Wang Meng and Chen Zhou

(*Fishing in Reclusion at Cha-hsi*, 2022)

(*Lofty Mount Lu, Lushan Gao Tu*, 2020)

Fast-forward a few centuries, I want to introduce you to a man called Richard Taylor. Taylor is the head of the physics department at the University of Oregon while also being involved in the departments of psychology and art. He has done a lot of research studying and analyzing fractals in different scientific scenarios and applications while also considering their relationship to art from a scientific point of view *(Taylor, n. d.)*.

One of his subjects was the work of Jackson Pollock. When analyzing many of Pollock's works, Taylor noticed that almost all of them exhibited fractal elements of various degrees and frequency. In fact, he even attributed the broader success of these otherwise abstract and often inaccessible or incomprehensible artworks to their fractal nature *(Taylor et al., 1999)*. This means that even if we are not particularly drawn to the motive of an artistic work, its use of fractal-like patterns can still have a subconscious influence on our minds. Our brains seem to be programmed to respond positively to fractal patterns both in art and in nature *(Hägerhäll et al., 2015)*.

Richard Taylor proceeded to calculate the fractal dimensions throughout the artistic series of Pollock and concluded that throughout Pollock's career, they constantly increased in factor. He also stated that it was possible to measure the authenticity of Pollock's works to a degree of 93% accuracy due to the analysis of their respective fractal structure *(Arthive, 2018)*.

Figure 23 on the next page shows some examples of Pollock's artworks with fractal patterns:

Figure 23, Fractal Paintings by Jackson Pollock

(*Mural*, 2021)

(*Konvergenz*, 2024)

Another famous artist that was already introduced in an earlier chapter who also intentionally explored the mathematical side of art and also experimented with fractals was Maurits Cornelis Escher. Besides his more well-known optical illusions, he drew mandala-like fractal shapes made out of psychedelic motives that seem to grow from and into endless reiterations of themselves. Figure 24 shows two examples:

Figure 24, "Circle limit with butterflies",
"Less and less", both by Escher

(*Circle Limit With Butterflies*, 2024)

(*Less and Less*, 2024)

It could be argued that fractals are nature's artistic approach to self-reference, much like we as artists only exist in the context of reference to each other. Much like nature, we are only able to be different towards each other particularly because we are at the same time so similar. Without being similar, we would not be able to recognize and acknowledge the aesthetics of diversity as difference in its purest form. We need to be similar in order to have the same understanding of difference and abstraction. Otherwise, everything would be the same in terms of not being distinguishable nor comprehensible. Unpredictability as an underlying structure of natural manifestation is what makes life exciting, and, at the same time, art itself so interesting. We know about the unknown and enjoy bathing in whatever separates itself from within our own minds or the chaos of nature. If something dares to distinguish itself against the broad canvas that is our world, we will be there to judge and remember. To recreate and dream the beauty of the flowers we have seen, which for us have reimagined their fractal nature buried deep inside their DNA into the sculptures they show themselves as. They want to be seen and we want to see. They are actors just like us.

As stated by him in his book *"The Fractalist: Memoir of a Scientific Maverick"*, Mandelbrot considered himself a "fractalist": "A challenge I kept encountering - one I never knew quite how to manage - was to do justice to the parts and the whole" *(p. 14)*. It was more than just a fancy theory to him. It comes as no surprise that it left an existential imprint on him and changed his view on reality itself. The undecidable principle of fractal geometry lead Mandelbrot to think of himself much less as a scientist or mathematician rather than what he defined as a "fractalist". This is his way of interpreting the tools that nature has given us in order to analyze the underlying principles of nature as incomplete when considering the vastness of the fractal realms. These realms, often unpredictable and undecidable per se, are our heritage of life and its origins as we know it and span their influence across the universe, no matter where we look.

Instead of differentiating order from disorder, one must accept, according to Mandelbrot's theories, that randomness and unpredictable variations are at the core of everything *(Mandelbrot, 2014)*. Chaos is what is spanning over both order and disorder; and that is why mathematics and physics fail in the fractalist world due to their constrained methods and language and the resulting simplified assumptions it can articulate. It is probably only through art that we can explore and contextualize what is hidden deep behind this curtain, and once again we find ourselves at the beginning of a road that leads into the vast unknown, which has yet to be discovered.

Geometric dreams

Walking the line

When imagining something infinite, most of the time our first instinct is to think about something linear instead of irrational numbers like pi. One such thing might be time itself. While Einstein has since proven in his famous theory of relativity that time is actually not linear at all, our intuition still paints the picture of a past, present, and future. We imagine all the things that "are" existing on a linear string of time. Thus, the concept of something being "infinitely long" (in terms of duration) would be considered by many as an idea that they can wrap their head around. The same happens with dimensions of different sizes. Something that is considered infinitely big can be also considered as something simply too big for us to ever measure. It is hard to imagine an entity without a physically conceivable boundary toward either of its dimensions.

Consider the term "infinite universe", for example. Of course it must seem infinte at first. No one knows for sure what is at the end of the universe, if it even has something like a conceivable end, or if it is just an ever-expanding dimension which can never have an end as we understand it *(Harvard Center for Astrophysics , n. d.)*. Even if we would somehow find out that it in fact could be measured, it would be impossible for beings like us to ever get to its boundaries because it most probably expands indefinitely at the speed of light. According to physics, we very likely might never be able to catch up and fling our measuring tape around it. Part of the reason is that the speed of light is also the physically agreed upon barrier for the maximum speed at which something with any kind of mass can move throughout the cosmos *(Petruzzello, n. d.)*. This means that even if it has an end, we would never be able to reach it and verify its existence. As Eric Betz writes in his aptly titled article "The Beginning to the End of the Universe: The Big Crunch vs. The Big Freeze", it is also still undecided if the initial impulse of energy from the Big Bang as the cause of the universe's presumed infinite expansion was either too little, just enough, or too much:

Too little: The universe would at some point stop expanding and the sheer mass of everything inside it would start to pull on it until it collapsed back into an infinitely small and dense point while crushing everything inside. This would most probably lead to another Big Bang in an infinite series of Big Bangs *(Betz, 2023)*.

Just enough: The universe would at some point be exactly balanced. The gravitational pull from the inside would equal the energy of expansion and it would simply stop growing at some point and stay exactly the same size forever. Eventually, everything would be sucked into black holes which would then be sucked into each other until there remained an eternal and completely empty universe *(Betz, 2023)*.

Too much: The universe would never stop expanding and thus everything within it would drift further apart forever. The distances between planets, stars, galaxies, clusters, and so on would never stop increasing and thus the energy needed to form new structures would be eventually evaporate and dilute into the incredible vastness of the cosmos. The remaining energy would also eventually be sucked into black holes, most probably never to return to our physical universe again. Everything would slowly become dark and approach absolute minimum temperature of −273.15 C° *(Betz, 2023)*.

The reason I wanted to introduce this point is to give ourselves a quick reminder that not even the fate of the cosmos is something we are able to predict with any degree of sufficient certainty. We might simply never be able to do so, either. What will eventually happen with what we know as time and space is undecided. But the good news is that we also do not have to worry much about it. The time it takes for either or none of these scenarios to occur is so inconceivably long that we might as well just call it infinity, grab our favorite beverage, and forget about it for now.

A much simpler example of something infinite we can try to imagine is something that is endless in only one direction - a line, for example. A simple line is one of the most rudimentary forms of geometry and is also the basic vector in which units like the metric system are applied to measure distances. So if we are bold enough, an endless distance can be imagined like an endless line.

Euclid's nightmare

When we think about mathematics in general, our minds starts to generate these images of numbers and complicated formulas which tortured us in school and can probably only *truly* be understood by those who pride themselves as mathematicians and physicists. This was not always the case. The truth is that the general understanding of mathematical concepts and their vast implications on our everyday lives started as a journey through shapes and forms, rather than numbers. At the early beginning of trying to truly formalize the system around 600 BC - 300 BC, people approached what we nowadays call mathematics in a non-numerical way: geometry *(Frithowulf, 2023)*. Geometry stands at the very base of concepts like ratios, constants, proportions, axioms, and so forth. In those times, a greek man named Euclid was at the center of it all, as he is considered the inventor of geometry as we know it today. Euclid wrote a treatise of 13 books called *The Elements* in which he started to define the basic rules and components of geometry. He also wrote down five geometric axioms, which are now called *Euclid's postulates (Joyce, n. d.-b)*. Without diving too deep into the complexity of his whole work, the first book starts with a series of definitions:

- Definition 1: A point is that which has no part.
- Definition 2: A line is breadthless length.
- Definition 3: The ends of a line are points.
- Definition 4: A straight line is a line which lies evenly with the points on itself.
- Definition 5: A surface is that which has length and breadth only.
- Definition 6: The edges of a surface are lines.
- Definition 7: A plane surface is a surface which lies evenly with the straight lines on itself. (Joyce, n. d.-b)

It is important to note that these definitions are verifiably true. He proceeds with articulating more of these "simple" truths which make up the basic building blocks of any geometric object inside Euclid's system. If we take a step back and look at the broader picture, it makes a lot of sense to develop a formal system like mathematics by first trying to understand the forms of the objects that surround us. At this stage, there is no real need for algebra yet. In order to find mathematical truths which can be applied in a scientific way, we do not need those numbers or algorithms. It is actually more often the other way around: Many of those seemingly mysterious numbers like pi, or the "golden number" phi have their origins in discovering the corresponding geometry beforehand.

The irrational numbers and complicated formulas are often naturally integrated in geometrical forms. Drawing a circle (given the right tools) or using circular forms is much more easy and intuitive than actually calculating one (as we have seen in a previous chapter). This is especially true for the earlier ages of mankind. To clarify, it was not necessary to understand the inner workings of the mathematical constants and properties of a circle in order to build a wheel to be attached to a horse chariot. And there is a lot that can be done to solve a problem with only geometrical methods without the need for a calculator.

> *An ancient writer said that arithmetic and geometry are the wings of mathematics; I believe one can say without speaking metaphorically that these two sciences are the foundation and essence of all the sciences which deal with quantity. Not only are they the foundation, they are also, as it were, the capstones; for, whenever a result has been arrived at, in order to use that result, it is necessary to translate it into numbers or into lines; to translate it into numbers requires the aid of arithmetic, to translate it into lines necessitates the use of geometry. (Lagrange, 1795)*

The "ancient writer" in the quote from Joseph Louis Lagrange is Plato, our trusted companion - here he comes again. In order to define something being true about something else, there always must be something reproducible involved in the process. In order for something to be reproducible, a system of pre-defined axioms is needed, for which we can then postulate a new logical system that must be inherently true, if the axioms it is based on are true. For geometry, Euclid defines five of these which he calls his *postulates:*

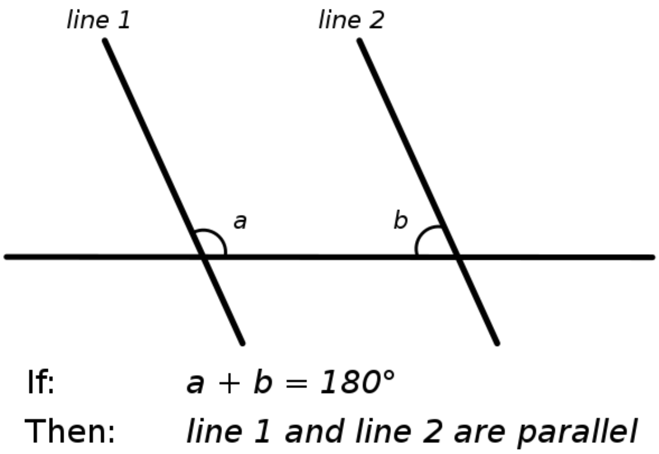
- Postulate 1: To draw a straight line from any point to any point.
- Postulate 2: To produce a finite straight line continuously in a straight line.
- Postulate 3: To describe a circle with any center and radius.
- Postulate 4: That all right angles equal one another.
- Postulate 5: That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles. (Joyce, n. d.-b)

In order for geometry to be a complete, consistent, and decidable system, all of these postulates must be true. And on paper, they literally are. But what if the paper and the lines are infinite?

The cursed fifth postulate

The fifth postulate of Euclid, also known as the parallel postulate, has been a topic of debate in the mathematical community for centuries and can be considered the ugly duckling when it comes to formalize geometry as a whole. While the other four postulates of Euclid's *Elements* seem straightforward and intuitive in the general perception of most, the fifth postulate has been viewed as less obvious and has led to various attempts to prove it or replace it with other postulates *(Szudzik & Weisstein, n. d.)*. This is because it can not be considered "true" in terms of being internally consistent like its four other siblings. So in order to convey why anybody could get so passionate about two parallel or non-parallel lines, I will briefly touch on the background and historical significance of it. Figure 25 on the next page shows a diagram of two parallel lines and what parallel actually means:

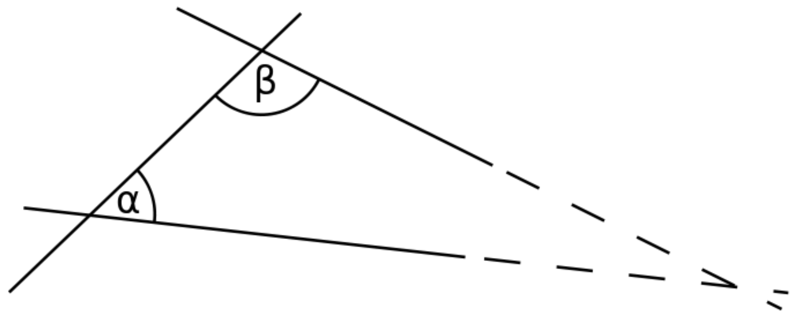
Figure 25, Diagram of 2 parallel lines



(*Diagram of 2 Parallel Lines*, 2024)

As mentioned before, Euclid's *Elements* is a mathematical treatise consisting of 13 books from about 300 BC that has had a significant impact on science as a whole, but particularly on the development of mathematics and its applications *(Joyce, n.d.-a)*. Each of these books deals with a specific area of geometry. While the first chapters of the *Elements* cover the basics of plane geometry, later on the book introduces the fifth postulate, which deals with parallel lines. And this is where it gets tricky: While it is actually not directly about two parallel lines in particular, it is about all the other lines and what happens if they are not parallel. Euclid's fifth postulate states that if a line intersects two other lines and the interior angles on the same side of the intersecting line add up to less than two right angles, then the two lines will eventually intersect on that side, as figure 26 on the next page demonstrates:

Figure 26, Diagram of intersecting lines



(*Diagram of Intersecting Lines*, 2024)

Alternatively, if the interior angles add up to more than two right angles, the two lines will never meet. This seems obvious at first, although there is a problem. The reason is that Euclid's other four postulates are what is called "self-evident" because they describe "truths" that can be proven in a very simple manner with the instructions given by them.

The fifth postulate, on the other hand, has been hotly debated since its introduction in the *Elements* and has been a matter of discussion for more than 2000 years now. In fact, it is the only postulate in Euclid's work that is not "self-evident" and, as mentioned before, this matter alone has led to countless attempts to prove it or replace it with alternative postulates by famous mathematicians like Adrien Marie Legendre, who dedicated a serious part of his life attempting to figure out a solution in his book *Elements of Geometry (Legendre, 1867)*. At this point, let us stop for a second and admire the beauty of something being self-evident or, as a matter of fact, inherently true. There is nothing falsifiable with the statement that one can draw a straight line between any point to any point. The absence of "self-evidence" in the case of the parallel postulate becomes apparent when infinitely smaller angles are introduced, for example. Imagine that you can make every delta of the sum of the angles needed in order for the two lines to meet at some point infinitely small or "infinitely divisible". This means that there are infinite iterations of how small the angle can be, just by infinitely dividing the remaining sum of the angle by half, for example, and doing it all again for the new sum of angles. This means that it is not provable within the given system of axioms that the two lines will ever meet throughout infinity while their combined angle on the inside still remains smaller than 180 °, because at the same time there are always infinitely more angles smaller than 180 ° *(Hamiss, 2019)*.

An approach from another angle, pun intended, would be to assume there are infinitely many points of possible intersection for the third straight line, meaning that if you extend the space between the first two lines, thus increasing the length of the third line throughout infinity, you can never verify the fifth postulate as true. Due to the infinite distance between the two lines that should intersect each other, their intersection would also only occur in an infinite space *(Hamiss, 2019)*.

So why, exactly, is that interesting? Why not just go the route of intuition and ignore the undecidability of this statement, and just assume that eventually, throughout infinity, they will meet anyway. Well, as you, my dear reader should suspect by now, there is a catch.

While the innumerable attempts throughout history of somehow finding a solution and proving or disproving the fifth postulate failed, mathematicians in the 19th century eventually encountered something even more elusive and equally fascinating by exploring the the possibilities of replacing the fifth postulate with other postulates, which would result in different types of geometry altogether. Welcome to the realm of non-Euclidean geometries *(Taimina & Henderson, 2024)*.

The first mathematician to do this was the Russian mathematician Nikolai Lobachevsky, who in the 19th century developed what is now known as hyperbolic geometry. Lobachevsky replaced the fifth postulate with another one that states that though a given point, there are multiple lines that do not intersect the given line. This postulate led to a geometry different from Euclidean geometry in that it contained no parallel lines and had many unique properties *(Singh, 2022).*

Later, in the mid-19th century, the Hungarian mathematician János Bolyai independently developed hyperbolic geometry, using a similar approach to Lobachevsky. At the same time, another mathematician, Carl Friedrich Gauss, was also exploring the concept of non-Euclidean geometries, but his work was not published until after his death.

Elliptic geometry, another type of non-Euclidean geometry, was independently developed by the German mathematician Bernhard Riemann in the mid-19th century. In elliptic geometry, the fifth postulate is replaced with a postulate that states that through a given point, there are no lines that do not intersect the given line. This leads to a geometry that is different from both Euclidean and hyperbolic geometry in that it contains no parallel lines and is finite *(Merriam, 2023)*.

To briefly summarize: The discovery of non-Euclidean geometries has been one of the most significant events in the history of mathematics, as it challenged the previously held assumption that Euclidean geometry was the only possible geometry. In fact, it showed that there can exist different kinds of geometry which are consistent while at the same time not relying on Euclids fifth postulate. The development of non-Euclidean geometries also had important implications for the study of physics and even significantly contributed to the development of the theory of general relativity by Albert Einstein in the early 20th century *(Ungar, 2005)*.

From an artistic perspective, I have focused on explaining these underlying principles to some degree of detail because I feel that it is often underestimated how intricately those lines, squares, and all other forms correlate with our understanding of the world and our artistic expressions. Most of the time, we take the objects that surround us in our lives for granted, without ever wondering if they exist on their own without any context to anything else, including ourselves. When we think about the material world, our mind does not struggle with giving it form. We imagine it as a divisible cosmos of smaller and smaller forms or, for a lack of better term, we imagine the material universe being made out of differently sized geometric forms and structures as a foundation. Building blocks in building blocks, often intertwined with fractal structures. As a consequence of this, we also use their representations intentionally and unintentionally in almost any form of visual artistic practice.

Even photography is, eventually and in its purest form, only the capturing of light from respective objects by decoding and imprinting their colors, which represent the visible wavelength of their reflected light *(Tate, n.d.-e)*. But even light itself can sometimes be both, a solid particle as well as a geometric pattern which we call a "wavelength" *(Piazza et al., 2015)*. Moreover, every distinguished object that wanders over the screens of our digitalized surroundings is most likely a pixel representation of a calculation of two- and three-dimensional forms that appear on the screen many times per second, giving the illusion of a three-dimensional space on a two-dimensional plane. It is an attempt to make the cyberspace decipherable for our human minds.

Geometric forms and language open the door to decoding the enigmas of art and reality and connecting science and creativity. Geometry holds the key to "making sense" of the world, both conceptually and artistically. This is why it must be considered the main building block of visual "algorithmic" or "generative" art. It is, in my view, the foundation on which visual context and visual artistic practice are built upon. Without a common denominator in accepting our perception of the forms around us as a collective and holistic experience, there would be no art, and there would be no context to anything else. Everything would be indistinguishable from anything else. It is a necessity for stepping into the realms of algorithmic or generative art. So what is the definition of generative art? The Tate Museum of Modern Art puts it like this: "Generative art is art made using a predetermined system that often includes an element of chance – is usually applied to computer based art" *(Tate, n.d.-c).*

So in addition to the shapes and forms that give visual context, generative art can be referred to art that is created through a process of algorithmic or procedural generation. Generative art therefore often involves the use of mathematical formulas and algorithms, which are intrinsically related to the process of discovering and exploring non-Euclidean geometries. They are needed to generate these shapes, patterns, and colors that make up the visual experience of any screen based digital media art. One of the most important persons in this discipline, probably *the* pioneer in this field and computer art in general, was the artist and scientist Herbert Franke *(ZKM, n.d.)*. The following chapter will provide a glimpse into his involvement and contributions to this genre and take a short trip through time, to the origins of computer art itself.

Automat und Mensch

Born in Vienna in 1927, Franke began his career as a physicist and science journalist before turning his attention to the burgeoning field of computer art. Over the course of his career, he produced a vast body of work that explored the intersection of art, science, and technology, influencing numerous artists and designers in the process. This very short introduction to his person must be seen as a mere understatement of what his contributions and engagement really meant for the world of digital media art. As we can deduct from the cornerstones of his biography, he can be defined as one of the creational pillars directly involved into putting media art into the broader perception and attention of the public throughout the 20th century *(ZKM, n.d.)*.

Franke's interest in computer art began in the early 1960s, when he first encountered computer-generated images while working as a science journalist. Fascinated by the possibilities of this new medium, he began experimenting with computer programming and digital imaging techniques himself at Siemens in 1952 *(Vasilieva, 2022)*. He is widely regarded as a very "hybrid" individual, trying to explore and bring together the fringes of what was and is possible in the intersection between human and machine. He was a dreamer, a visionary, an artist, and a theoretical physicist. In an interview for the Kate Vass Galerie, he is described as a "border-crosser" between science and art:

> *Herbert W. Franke is in many ways a border-crosser between science and art: He is a scientist (a physicist and speleologist), artist (in the visual arts and as an author) and art theorist. (Vasilieva, 2022)*

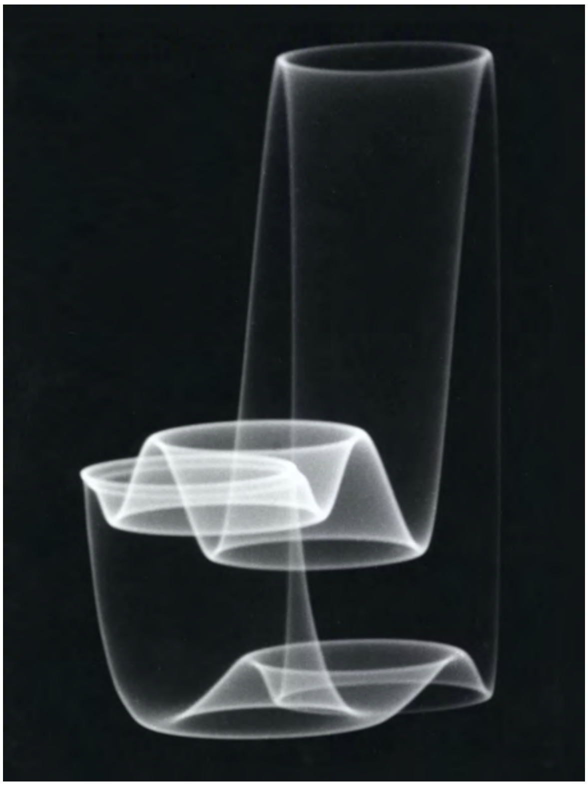
It is also precisely this intersection that is interesting to me in the context of this paper. While Franke was curious by nature, I think it is fair to say that what actually defined his career was a kind of naturally emergent process that combined his talent and creative potential with post-war technologies and their newly discovered potential to be used as an extension to the artistic mind. As with other artistic fields, science is often only one step behind as an underlying structure of analyzing the world and identifying the kinds of problems that we want to engage in an artistic way.

> *My idea of art was and still is: to venture into unknown new territory. Perfection or purist ideas could not be in the foreground, I was always driven by curiosity. I wanted to fathom what and why a machine can do this or that particularly well. (Kent, 2022)*

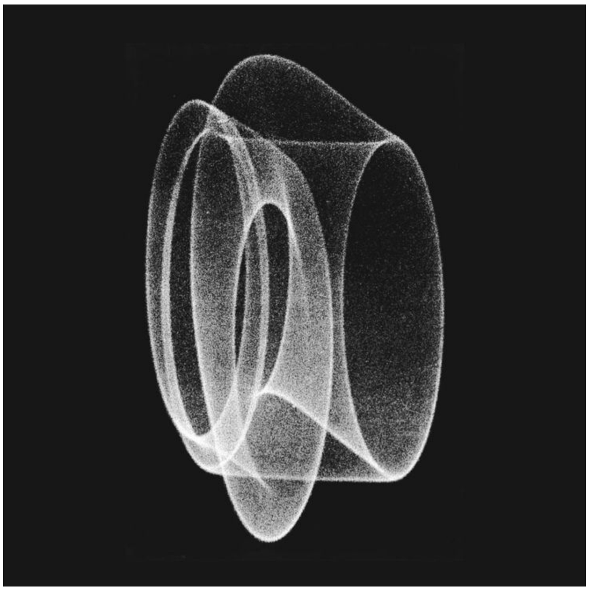
The above idea gets clearer when we start to look at one of his earlier works which originated from his employment and access to technical equipment at Siemens. It is called *Dance of Electrons*.

Please take a look at figure 27 on the next page:

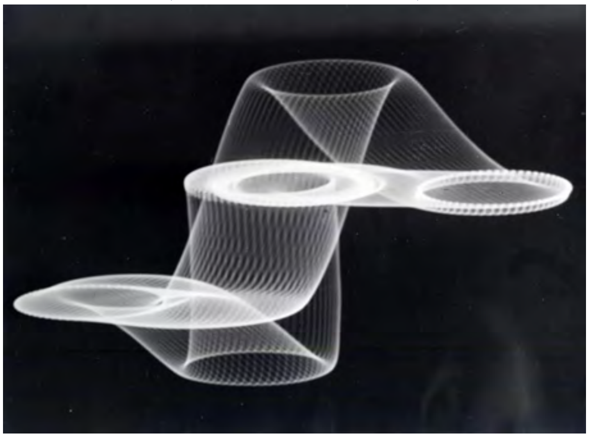
Figure 27, "Dance of Electrons"
by Herbert Franke



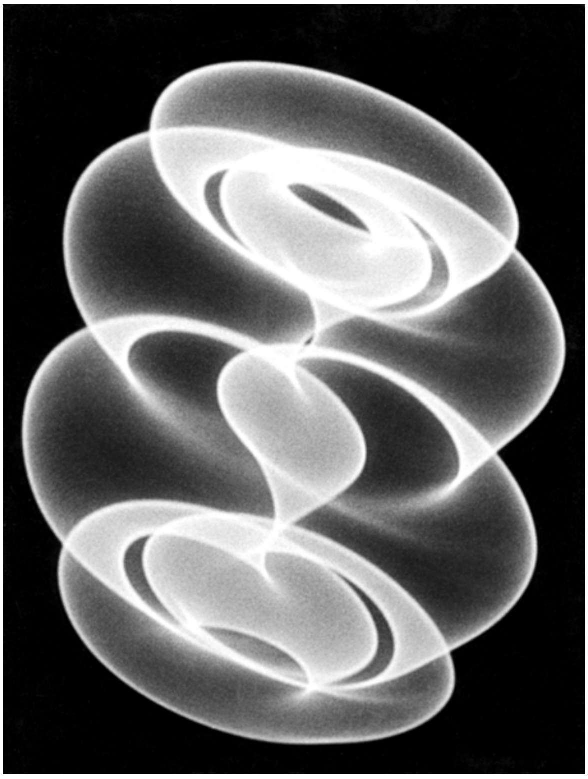
(*Dance of Electrons*, 2021)



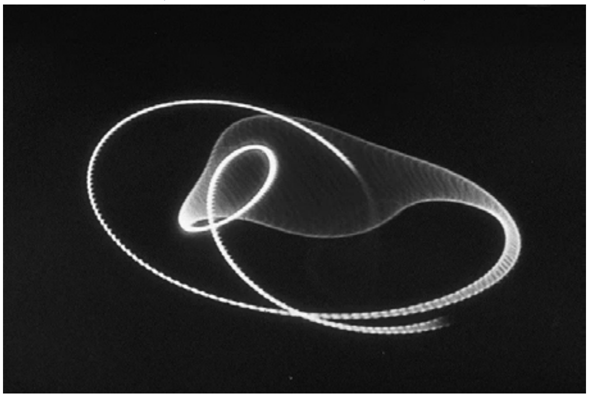
(*Dance of Electrons*, 2009)



(*Dance of Electrons*, 2021b)



(*Dance of Electrons*, 2024)



(*Dance of Electrons*, 2015)

The images in figure 27 were created by programming an analogue computer to output certain patterns of waves to an oscillograph and putting a camera in front of it to take a picture with an open aperture *(Kent, 2022)*. What is interesting here is the intention of transforming something ultimately theoretical in its inherent attributes, which is the electron, or many of them, respectively, into something that describes a form. This work is a visualization of electrons hitting a fluorescent screen that then emits light as a reaction towards the camera or the observer. The forms that evolve throughout the timespan of a picture are the pirouettes of these electrons "dancing" in the space-time-continuum.

> *From the beginning, I worked with electrons, that is, with light, and I wanted to design it in such a way that art would emerge from it. This was the fifties and generative art. Once I had digital computers at my disposal, I also had the desire to produce art that could be quantified in terms of information theory. (Kent, 2022)*

The above quote shows how Franke makes a clear distinction between this being one of his earlier works without having access to digital computers yet and his later works which incorporate more modern methods of producing generative art. *Dance of Electrons* is completely analog, which allows an interesting conclusion about computer art in general. In its beginning, computer art was analog.

Cellular automata

To follow up into the digital realms of computer art I want to discuss one of Franke's later artworks, called *Cellular Automata*. The term itself is actually borrowed from the field of simulation theory and describes a mathematical process where a pre-defined entity or "cell" is changing its own state as well as influencing the state of other cells in a closed system by evolving throughout given intervals of time via a finite set of rules. These kinds of models are also known as emergent or evolving systems. They have been used by artists and scientists alike to create patterns and models in different disciplines to research and find answers to certain problems that are hard to solve without algorithmic automation. While being mathematical in nature regarding their internal structure, the process of cellular automata is often visualized to give it context. The visualizations are very often the creation of artists and aesthetic enthusiasts of abstract ideas; and their often mesmerizing patterns can be a very satisfying artistic experience to watch. But before diving into the artistic aspects of evolving systems, let us first take a look at how the *Stanford Encyclopedia of Philosophy* defines the term "Cellular automata":

> *Cellular automata (henceforth: CA) are discrete, abstract computational systems that have proved useful both as general models of complexity and as more specific representations of non-linear dynamics in a variety of scientific fields. Firstly, CA are (typically) spatially and temporally discrete: they are composed of a finite or denumerable set of homogeneous, simple units, the atoms or cells. At each time unit, the cells instantiate one of a finite set of states. They evolve in parallel at discrete time steps, following state update functions or dynamical transition rules: the update of a cell state obtains by taking into account the states of cells in its local neighborhood (there are, therefore, no actions at a distance). Secondly, CA are abstract: they can be specified in purely mathematical terms and physical structures can implement them. Thirdly, CA are computational systems: they can compute functions and solve algorithmic problems. Despite functioning in a different way from traditional, Turing machine-like devices, CA with suitable rules can emulate a universal Turing machine (...) and therefore compute, given Turing's thesis (...) anything computable. (Francesco & Tagliabue, 2023)*

To add a little context to this definition: The origins of this concept are founded within the academic world of Los Alamos National Laboratory in the 1940s and were first discovered by Stanislav Ulam and John von Neumann. While initially only being studied mostly behind the thick walls of academic obscurity, it was Stephen Conway's well-known adaption of these principles named *Game of Life* which made it a topic of interest for a broader audience *(Johnston, n.d.)*. For this paper, it is important to indicate that the premise of these kinds of systems is always the same: To generate and simulate complex and emergent behaviors and patterns which stem from relatively simple conditions or sets of rules that are set prior to running the simulation. Lex Fridman, a Russian-American scientist well known for his podcast *The Lex Fridman Podcast,* notes in one of his shows that in relation to the works of Stephen Wolfram, a British-American computer scientist and expert on complex systems and cellular automata, cellular automation as a process can be seen as a discreet *theory of everything*:

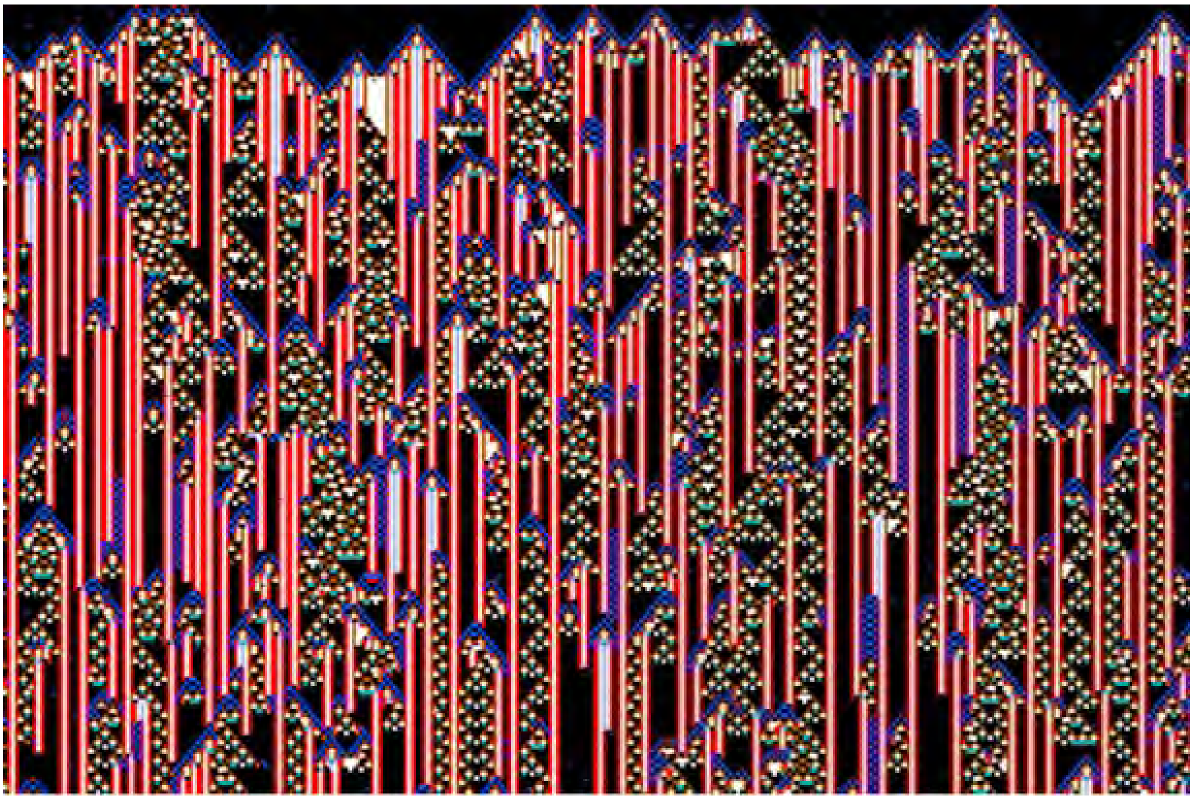
> *From simple rules and simple objects and hypographs emerges all of our reality. Time and space are emergent and basically everything around us is emergent. (Lex Clips, 2021)*

This is an interesting thought, since it allows the perspective of deducting cellular automata as some kind of representation or abstract model of the fundamental behavior of everything in the cosmos. It hints at a relation between everything only existing and evolving because of a very basic, yet unknown set of deterministic rules. Once set in motion, these simple rules lead to everything that is and will ever be. Of course, this is only true if reality is made out of some kind of basic building blocks that are able to follow these rules and are able to evolve to more complex structures, which always was and still is up for debate in modern physics to this very moment. Moreover, in this case, the answer begs the question if reality is or is not following a deterministic set of rules; and, no matter the quality of the argument, either of the two possibilities might never be verifiable, making it an undecidable system in itself *(Hoefer, 2024)*.

Due to their evolving nature, cellular automata also inherit many intrinsic properties of algorithmic sequences. One of the most important ones in the context of this paper is clearly the undecidability of an evolving system throughout time. Many of these systems, including Conway's *Game of Life* can be considered undecidable systems themselves, since there are configurations of evolving patterns within the simulation where it is impossible to predict the outcome of the simulation throughout a finite span of time. What is interesting in this particular case is that these scenarios can occur along with "complete" or so called "turing complete" scenarios where the cells eventually all die or are caught into an endless loop of self-reference *(Wikipedia contributors, 2024b)*. This means that the same set of rules for the simulation, if run separately or in multiple iterations, can produce determinable outcomes while simultaneously producing patterns that are evolving in a way that makes it impossible to predict if the system will ever halt, get stuck in a loop, or continue evolving throughout infinity. That is also why Herbert Franke was so immensely fascinated by this topic and named a series of artworks after them.

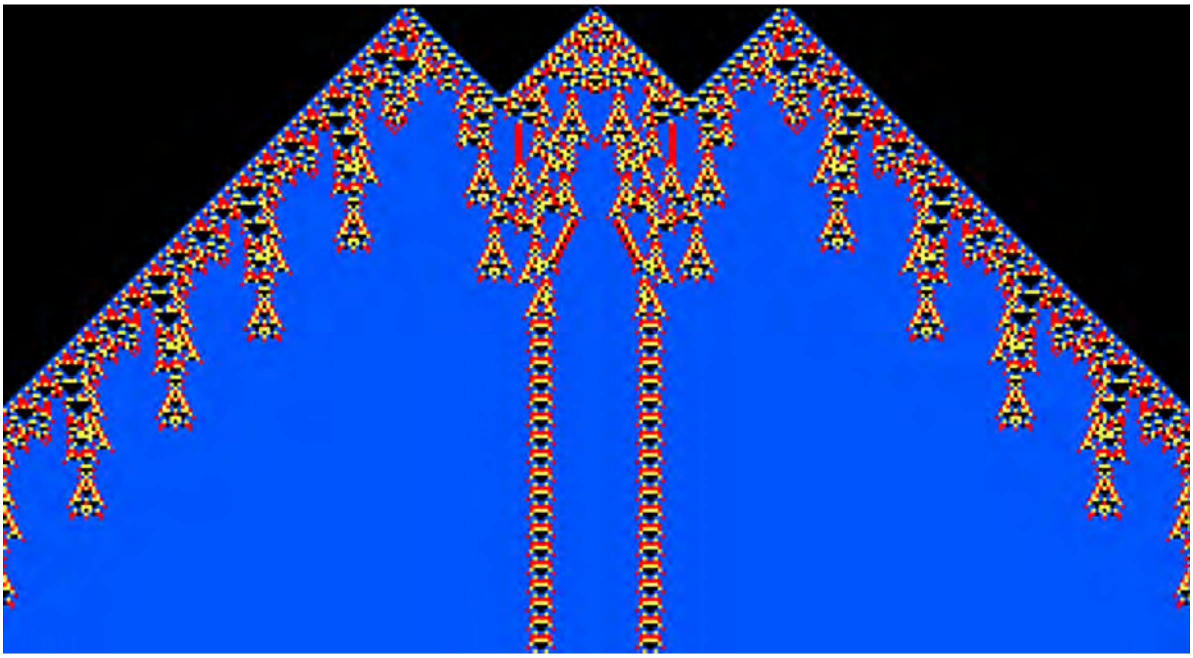
Figure 28 displays screenshots from Franke's dynamic *Cellular Automata* simulations:

Figure 28, Cellular Automata by Herbert Franke



(*Cellular Automata*, 2024)

Figure 28, Cellular Automata by Herbert Franke



(*Cellular Automata*, 2024b)

Herbert Franke was certainly a pioneer of his crafts and a visionary of his time, allowing us to see art and science relative to each other in a different context. They are much more intertwined and familiar than might be expected at first glance, and also occasionally love taking a stroll together on the promenade outside of Plato's arcade. Artists like Herbert Franke invite us to come along for a trip we could not have imagined before. So I want to conclude with Herbert Franke's rather clinical approach:

> *Art is always about mathematics. Every image can be described mathematically. This also applies to music, take, for example, the theory of harmony in music. There are examples from the history of art – the so-called 'golden ratio', which has its origins in ancient Greece and has always been considered the essence of aesthetics. There is nothing more than a formula behind it. Human perception is a process of data processing. We analyze, filter and encode all information that affects us. Above all, filtering is crucial because we have only limited processing capabilities. (Vass, 2022)*

Fast forward to the year 2024, cellular automation has become an invaluable toolset in many scientific disciplines since it allows us to make predictions about evolving systems in nature which are too complex to compute otherwise *(Quantum News, 2024).* It is successfully used in modern research and has also influenced the field of computer arts immensely. In addition to Herbert Franke's contributions to computer art, he was also a prolific photographer and filmmaker, and his work in these fields often explored similar themes of science, technology, and the natural world. This interdisciplinary approach to art and science helped to bridge the gap between these two fields, which transformed him into a key figure in the development of a new kind of artistic practice that was heavily influenced by emerging technologies at that time.

Cubistic awakening

From a materialistic perspective of the world, geometry serves as the baseline of giving objects "form". Experiencing these forms always happens from an arbitrary perspective. For us as human beings, it is not possible to observe the world around us from multiple perspectives at once. This is also why up until the 20th century, art would most likely depict a representation or reimagination of these forms and from singular perspectives *(Janson, n.d.)*. Let us take sculptures as an example. Although they are made to be experienced from any possible angle, their forms can only ever be experienced from one single perspective at any given moment. At the same time, this single point of view is enough for giving the brain context related to all the other possible perspectives you are not seeing. If we look at an object from the front and can recognize its general concept (e. g. if we identify a round fruit as an apple), our brain is then able to puzzle together all the properties of its entire form and character. We know or can at least imagine how the back of the object should look like, according to our experience interacting with other entities of the same category in the past.

But still, what we see at any given moment in time is only one representative angle of the apple, not the apple itself. And although you can look at a painting from different angles, the objects you see in the painting have been arranged to simulate this same single angled approach to perspective, much as if you would look at the real world. The artist has made the decision from where to experience its entities from prior to another person looking at it. You, as the observer, are put into your intended space.

In the beginning of the 20th century, two men set out on an adventure intending to change that: Georges Braque and Pablo Picasso. They did nothing less than to invent cubism *(Tate, n.d.-b)*, an art style which was pretty scandalous for the renowned art communities at that time. At the *Salon d'Automne* in 1908 in Paris, the 26-year-old Georges Braque crashed into the scene by boldly presenting some of his latest paintings for submission to an upcoming important art exhibition. One of the critics in the committee to judge the submissions was the famous and well-respected painter Henri Matisse. When he took a first look at the paintings of Braque, he was shocked and replied in a disdainful manner that he thought these works were nothing but absurd *(Trachtman, 2013)*.

"They are made of little cubes!", Matisse said and voted to reject them for submission *(Trachtman, 2013)*. From then on, this kind of style was known as *cubism*. Braque was a close friend of Pablo Picasso's, and they both were inspired by the avant-garde and adventurous spirit of their time. One would not easily guess the inspiration/idols of these still young artists. It were the Wright brothers, inventors of the flying machine *(Metropolitan Museum of Art, n. d.)*. Their boldness in so radically breaking with artistic traditions was much more driven by their fascination for conquering the world through inventions rather than being artistically provocative. It was more a side outcome of what they were trying to do. Famously, the Wright brothers were by far the biggest contributors to the cause of getting humankind airborne. They built the first working planes and were pioneers in aviation and invention.

That is also what Picasso and Braque were bound to do. They wanted to reinvent art. They wanted to reinvent perspective. They wanted to free themselves and their work of the constraints that contemporary artists were, more often than not, afraid to trespass *(Rewald, 2004)*. The idea was simple, but incredibly powerful. It was in fact so powerful that it completely changed the dialogue of artistic practices forever. They noticed that, as noted above, if you look at any object around you,

you are always only able to see it from a single point of view. So they came up with the idea that, instead of painting things "as you see them", it would be much more interesting to paint things as would could *imagine* them. So they started imagining objects and their form from many sides and perspectives, flattened their surfaces, curved their straight lines, and generally used geometric language to describe hint at what you are looking at rather than showing you directly. They let go of your hand as soon as you are trying to grasp the concept of their art and would wait for you in the opposite corner of the room with a warm but cheeky smile. They were deconstructors of reality itself. As a homage to what inspired them in the first place, they also produced some works conceptualizing aviation and celebrating many aspects of the modern world.

Figure 29 is a cubistic interpretation of an airplane:

Figure 29, Cubistic Interpretation of an Airplane

(*The Scallop Shell: Notre Avenir Est Dans L'Air*, 2014)

There are three different types subgenres of cubism, *(Tate, n.d.-c)* one of which I want to highlight in particular. It is called analytic cubism. It got its name from its artistic method: The artist dissects an object into many single parts and then merges them back together from completely disconnected angles, as if the human perspective was no longer valid.

> *It is termed analytical cubism because of its structured dissection of the subject, viewpoint-by-viewpoint, resulting in a fragmentary image of multiple viewpoints and overlapping planes. Other distinguishing features of analytical cubism were a simplified palette of colours, so the viewer was not distracted from the structure of the form, and the density of the image at the centre of the canvas. (Tate, n.d.)*

This perspective encourages us to imagine objects not as concrete, but as a dissolved multitude of their properties all at once, only manifesting themselves and admitting to cohesion when we "choose" to take part in a perspective. This approach is an attempt to deconstruct the physical world into its fundamental properties and basic forms. It challenges the idea/practice of allowing objects to maintain a stringent identity that is projected outward towards a potential observer. It takes away the need for art to be concrete. In my view, it is a way of leaving the world incomplete and thus undecided. The single pieces are more than their sum.

Figure 30 on the next page, for example, shows a glass on a table:

Figure 30: Cubistic Interpretation of a Glass on a Table

(*Glass on a Table*, 2024)

To a degree, cubism is a product of its time and thus encapsulates the spirit of the dawning modern era, celebrating invention and exploring a new way of looking at things. It might not be a coincidence that at the same time, a relatively similar train of thought was taking shape in the scientific world of Central Europe. It was the beginning of a long venture into the unknown. The first steps of mankind to conquer the abyss of learning more about the fabrics of reality. It was the birth of quantum theory. Once again, while at first it seems that art and science are sometimes worlds apart in what they are trying to achieve, a closer look often reveals that the conclusions they draw end up at common ground:

> *Cubism and quantum theory both moved away from the idea that the world is representable from a single perspective. In the first decades of the twentieth century, it is the whole of European culture that no longer thinks we can represent the world in a simple and complete way. The anthropologist Claude Lévi-Strauss understands that to study a culture is to change it. (Rovelli, 2021, p. 67)*

In the 18th century, a Presbyterian minister named Thomas Bayes studied probability. In the early years, when quantum theory was slowly gaining traction, a new way of thinking was coined after him: "Quantum Bayesianism", or in short, *Qbism*. If read out loud it sounds exactly like "cubism". This is of course no coincidence by any means.

> *[T]he word 'QBism' alludes also to the Cubism of artists such as Georges Braque and Pablo Picasso--the influential style of painting that was shaped in Europe in the same years that quantum theory was developing. (Rovelli, 2021, p. 67)*

Qbism is the idea that, once and for all, we have to abandon a realistic image of the world beyond what is observable. It alters the conception of science as a tool for "measuring" quantitative properties of the world. Instead, the predictive qualities of science become only relevant subjectively in the context of absolute knowledge. Without an observer and valid observables, nothing can ever be concluded. The world is always only reflected in the observer, including the methods of observing, like art or science.

> *It also provides us with a vision of reality, a conceptual framework for thinking about things. It is this aspiration that has made scientific thought so effective. (Rovelli, 2021, p. 68)*

This radical idea leads down a path where we have to ask ourselves if the tools we have been using to establish order in the chaos of understanding, might themselves be subject to a relative perspective. Are the parts that make up a seemingly unshakeable, consistent, complete, and decidable system like mathematics maybe telling a different story if they are separated from each other and inspected? Is there more to find than their cumulative whole reveals at the first look? To answer this question, we have to dissect mathematics into an imaginary cubistic painting - look at its components from different angles and perspectives, flatten out its form and look beyond its cold symbolic DNA made out complicated formulas, numbers, and deterministic ideas.

We must know, we will know

Ignorabimus

At the end of the 19th century, a boy by the name David Hilbert is born in Königsberg in the kingdom of Prussia. It is him who will become one of the most important and influential mathematicians of the late 19th and the early 20th century. He became famous for his involvement in discovering and developing many of the fundamental ideas in different ares of mathematical theories and problems *(Kaplansky, 2024)*.

Hilbert's true obsession, though, was the very nature of mathematics itself. Following a similar path like Bertrand Russell and Gottlob Frege, he was trying to find a way to use formal systems constrained in and described by agreed-upon sets of axioms in order to define the necessary circumstances for a logical system like mathematics to exist in the first place:

> *No more than any other science can mathematics be founded by logic alone; rather, as a condition for the use of logical inferences and the performance of logical operations, something must already be given to us in our faculty of representation, certain extra-logical concrete objects that are intuitively present as immediate experience prior to in thought. If logical inference is to be reliable, it must be possible to survey these objects completely in all their parts, and the fact that they occur, that they differ from one another, and that they follow each other, or are concatenated, is immediate, given intuitively, together with the objects, is something that neither can be reduced to anything else nor requires reduction. This is the basic philosophical position that I regard as requisite for mathematics and, in general, for all scientific thinking, understanding, and communication. (Hilbert, 1996, pp. 228 f.)*

Hilbert's approach shows clear parallels to that of fellow Königsberg native Immanuel Kant, specifically his conceptualization of *a priori* knowledge *(Kant,* *1919)*. His point is that systems of logic, like mathematics, cannot be viewed as independent from immediate, sensory perceptions. Scientific knowledge of any kind is inextricably connected to our own limited and very human way of being able to perceive the world around us. This poses the question if logic and mathematics are sufficient means to find answers to our questions, or if we need to start asking questions about our methods themselves to find out if we are on the right paths in our pursuits of finding or approaching objective truth. This is why it is inevitable to also think about science in a philosophical and artistic way by constructing the necessary framework science can exist in.

This quasi-dilemma lead Hilbert to formulating three major questions amongst many other he thought would unveil more about the DNA of mathematics as a system *(Hilbert, 1902)*:

- Is mathematics a complete system? Meaning is there a way to prove every true statement.

- Is mathematics a consistent system? Meaning is it free of contradicting itself ie. can there be simultaneously a proof for A and NOT A at the same time.

- Is mathematics a decidable system? Meaning is there an algorithm which can always determine if a statement follows its given axioms.

Hilbert was convinced that all of these questions can be answered and that the answer would always be: Yes.

> *In opposition to the foolish Ignorabimus our slogan shall be: We must know - We will know. (Kühner, 2018)*

And he was wrong.

Kurt Gödel says no.

It might be an understatement to say that mathematicians got very nervous about these fundamental and unanswered questions Hilbert proposed. Up until that point, it was a very popular opinion to assume that we would be able to decipher the code of the universe itself, if we only tried hard enough and used the right tools. One of the most important tools, of course, being mathematics in a broader sense. Until then, even the great physicist Albert Einstein (amongst many others) was convinced that there must exist something equivalent to a "God's formula", or a "god's mind" *(Journeyman Pictures, 2021)*. In this context the word "god" is used as a synonym of everything existing rather than an omnipotent being. For the sake of understanding and also the linguistic spirit of the time, "God" equates to "Everything" in the realm of german scientific communities in the early 20th century. That is also the root of one of his most famous quotes:

> *I believe in Spinoza's God, who reveals himself in the lawful harmony of all that exists, but not in a God who concerns himself with the fate and the doings of mankind. (Gallagher, 2023)*

This is the quote Einstein used when answering the question whether he believed in a god or not. His semantic of the word "religious", reflected by his realist stance, was:

> *I have no better expression than the term "religious" for this trust in the rational character of reality and in its being accessible, at least to some extent, to human reason. (Baggott, 2024)*

Both of these quotes combined paint a good picture about what is necessary to construct the philosophical framework in which the arguments were made on a more mathematical basis. Einstein had a deterministic view of the universe. He was convinced that there was no such thing as free will. He was convinced that "God's lawful harmony" was strictly governed by the physics of cause and effect *(Gallagher, 2023)*. That means that there really is no room for an incomplete system. Thus every system that exists, even if not comprehendible or unknown to us, must not be undecidable. That does not mean that it has to be decidable for us. But it must be coherent in its inner logic, it must be a predetermined puzzle piece of the whole composition.

Let me give you an example: Our already trusty comrade, the irrational constant pi for instance. For us, it is and might forever be an undecidable system. Though we know exactly how to calculate every single digit of it, we can not say if or when it ends. This would mean that we could spend a literal eternity trying to calculate it and would never come to a true conclusion, although everything we have done upon to an arbitrary point in time still remains "correct". But following Einstein's logic, even the number pi, in the context of determinism, must be a complete number, an inherently whole piece of the mechanism. And now it gets complicated:

The above, of course, can only be true if the systems we use to even identify this part of the puzzle (pi) and the methods we use to solve the equation are at the same time also a valid part of the puzzle itself. Meaning the whole system is "complete". So if mathematics itself is not a "complete system", this would be a real problem. It would mean that pi as such might not even be the right answer to not even the right question which is the formula of pi itself. Not even the axioms which lead to pi might be "true" in a deterministic way.

So, is mathematics a complete system?

That is where Kurt Gödel comes into play. Similar to Hilbert, he also was a very important figure in the scientific world of the 20th century. Gödel also had a massive influence in terms of scientific breakthroughs in the fields of logics, mathematics, and philosophy. Some of his colleagues at the time where Gottlob Frege, Bertrand Russel, David Hilbert, and Georg Canter, just to name a few *(Institute for Advanced Study, 2021)*.

At that time, he and his colleagues were trying to use logics and axioms in order to explore and understand the foundations of mathematics as a whole. Following the questions Hilbert had formulated previously, Gödel (amongst others) was determined to find an answer to each of them. He developed a system to disprove that the common conception of mathematics as a complete system was wrong *(In-*

stitute for Advanced Study, 2021). The "Gödel incompleteness theorem" is a very complex process of encoding and decoding algorithmic operations and assigning natural numbers >=0 to different operations *(Raatikainen, 2020)*. It is known as one of the most profound and most impactful research efforts in the field of mathematics in the 20th century and had a devastating impact to the formalist movement which David Hilbert belonged to. The following is an attempt to simplify these matters and extract the logical implications I consider relevant for this paper. I will try to keep it simple:

Imagine you could assign a natural number to every possible symbol or combination of a logical or mathematical operation. For example, the number zero (0) gets the "ID" 6. The symbol "=" gets the "ID" 5. So a simple true formula would be: 0=0. Zero equals zero. If we write the same equation with the numbers assigned in our new system, it would look like this: 656. 6 represents the natural number 0, 5 represents the "=" symbol, and the 6 at the end a 0 again.

Now, in order to give the whole equation "0=0" its own ID inside the system, Gödel invented what is called the "Gödel number". This works by inserting prime numbers between all the "ID"s starting with the prime number 2. So our 0=0 aka 656 becomes 2 to the power of 6 multiplied with 3 to the power of 5 multiplied with 5 to the power of 6:

(2^6)*(3^5)*(5^6) = 243 000 000.

So the Gödel number for the mathematical expression "0=0" is 243000000. At first, this seems like an overly complicated system, but is important for what comes next. With this cumbersome system, you could assign a Gödel number to any arbitrary mathematical or logical expression and it would always be unique. You could also do a prime factorization on it and backtrack what symbols are encoded in any of the Gödel numbers. In this way, it is possible to encode both true statements (e. g. 0=0) and false statements (e. g. 0=1). Now what it thus also enables is to get Gödel numbers for axioms and their proof; an axiom being a principle that is considered inherently true without proof while also not being provable.

Because the Gödel numbers of more complex formulations would become extremely long and impractical, Gödel allowed to assign letters to this numbers as well. So instead of writing 243 000 000 you could only write "a" for example. Now comes the fun part:

Somewhere in this set of a metaphorical deck of cards containing their respective Gödel numbers there is a card which postulates the following logical statement: "There is no proof for the statement with the Gödel number G." Now of course, this card has its own assigned Gödel number, which is "G". But is that even possible? What does that even mean?

This card basically says that its own statement - and thus, the card itself - is unprovable. In other words, "this statement is unprovable", meaning there is no possible combination of cards inside the whole set which can prove this statement. But is that even true? Just think about it. If you could prove that this statement is true, it would mean it is not unprovable. The consequence is that it would become provable by definition. But if it was provable by definition, the statement that there is no proof for itself would be proven. Which means it would create an unsolvable contradiction in itself. What has to be concluded in that particular case is that a - in this case - mathematical system with an unsolvable inherent contradiction must be considered "inconsisten".

The other possibility is that it actually *is* true that there is no proof within the whole mathematical system for "the statement with the Gödel number G". However, the consequence of that would be that you have a system with true statements in it for witch there is no proof.

This is more or less a more complicated and mathematical approach to the famous liar's paradox (and a close relative to the already touched upon principle of a "logic bomb") which goes like this: A liar is declaring: "I am always lying." So if he is lying when declaring that statement it means he is actually telling the truth. Which means he also just lied and so on.

What has to be concluded in that particular case is that a (mathematical) system with true statements in itself which have no proof inside the system is considered "incomplete". Truth and provability are not the same thing and can be disconnected from each other.

To summarize: Mathematics as a system could either contain true, inherently unprovable statements, making it an *incomplete* system; or the system allows for a proof of a statement which has no proof, which would make it *inconsistent*.

This is how Gödel's incompleteness theorem showed that mathematics as we know it is inherently an incomplete and/or inconsistent system *(Raatikainen, 2020)*. This means that there will always be things which might be true but never provable. This understanding sent a large shock wave through the scientific community, as it was always believed that mathematics was a very reliable tool for investigating, analyzing, and verifying subjects of interest though the lens of logic.

This leaves only one of Hilbert's three big questions: Is mathematics decidable? This is where Alan Turing comes into play. In 1936, Turing worked on an idea that would later be known as the *Turing machine*. In essence, the Turing machine can be described as the invention and imagination of a modern computer, purely based on theory *(Rodrigues, 2024)*. It is an abstract machine consisting of two main components. The first is an infinitely long tape with discrete segments, each of which can hold one symbol from a fixed, limited set. The other component is a mechanism resembling a "read/write"-head, which can read the symbol of a segment, or write to exchange it with another symbol. This "head" has a finite number of configurable states, which can be understood as its programming. The respective states dictate how to proceed when the head moves over a new symbol. The value of a segment determines which operation to execute *(De Mol, 2018)*. Based on the symbol, possible operations are a) writing a new symbol into a segment; b) moving one segment to the left or right; or c) halting the whole computation process.

The Turing machine is a very simple representation of a modern computer model. Back then, the technology needed to actually build such an apparatus was insufficient in many respects. While in recent history, people have constructed physical Turing machines, these can be understood rather as a proof of concept or a sculpture of the theory itself. Alan Turing used this abstract representation of an actual machine to prove a point in logic, rather than having any plans to build an actual computer. Nevertheless, while slow and cumbersome, a physical Turing machine would theoretically be capable of implementing any existing computer algorithm *(Rodrigues, 2024)*. But what what was it that Turing wanted to prove?

A halting problem

> *Sir, a well-known piece of folk-lore among programmers holds that it is impossible to write a program which can examine any other program and tell, in every case, if it will terminate or get into a closed loop when it is run. (Strachey, 1965, p. 313)*

What Alan Turing was interested in was a logical proposition called the "halting problem". Formulated as a question, it would go something like this: Given a computer program and an input, will the program terminate or run forever *(Halting Problem | Brilliant Math & Science Wiki, n.d.)*?

While this may at first seem trivial, it is anything but: It is a problem proven to be undecidable. While the logic behind it takes some time to grasp, it can be considered what is called a *supertask*. A supertask is defined as an uncountable series of infinite tasks *(Manchak & Roberts, 2022)*. To better demonstrate, I will introduce a concrete example of a supertask: *Thompson's lamp*. Consider a lamp switch which you can turn on or off. The time it takes to switch from one state to the other can be any interval, and it can be turned on and off an unlimited amount of times within a finite timespan. The underlying rule is that whatever position the lamp's switch is in, it must always be switched to the opposing state and cannot stay the same. With each switch, the switching-interval is divided by half. Imagine a sequence like this: The lamp starts with the switch 'on', the first switch to 'off' occurs after one minute. The next switch (to 'on') occurs at 30 seconds, then at 15 seconds (to 'off'), et cetera. In which state will the lamp be after five minutes? Will it be on or off?

As you might already have suspected, the answer to that problem is undecidable, because an infinite number of tasks must be completed to arrive at a definite outcome. Halving the interval of switching with each change of state means that the number of times the switch is flipped becomes infinite, even within a finite timespan of five minutes. There can never be a definitive state at the five-minute-mark, because even the smallest in-between interval before precisely five minutes can be divided ad infinitum. To clarify, the lamp is neither 'on' nor 'off' after five minutes, because it is in an infinite state of intrinsically solving the question. There is simply no answer to the problem *(Manchak & Roberts, 2022)*. The halting problem resembles this in its underlying principle.

As described in greater scope in Turing (1937), one can imagine a computer program (A) that is able to determine if the outcome of a given operation (c) will halt (=true) or run forever (false), based on its alleged (?) input (x). Then, this program takes as an input another program (B) that is able to reference the prediction of the base program (A) and always outputs the exact opposite of what the base program A predicts and feeds it back as input into program A. This means that because program A predicts an outcome based on its input from program B, B will automatically self-reference that prediction and feed the opposite into A, leading A to change the prediction leading B to change its output. This operation is repeated ad infinitum.

> *I propose, therefore, to show that there can be no general process for determining whether a given formula U of the functional calculus K is provable, i.e. that there can be no machine which, supplied with any one U of these formulae, will eventually say whether U is provable. (Turing, 1937, p. 259)*

To summarize: The question whether this program will ever halt or not is undecidable. While this might seem nonsensical at first for any real-world applications, it truly is not: There exist a vast number of real-world problems, especially in computer science, which are as of now considered unsolvable due to the exact same logic as the halting problem. If we would be able to solve one of these real-world "twins" of the halting problem, we would have reverse-engineered and solved them all at the same time.

A little Tinguely

The concept of non-deterministic outcomes and evolving behaviors has also found its way into the artistic world. In order for something being simultaneously observable and undecidable, by default, it has to be *non-conservable* at a present moment in its holistic finite state. For example, when an artwork explores undecidability through intentionally "not having a finite state in the moment", the artwork itself can at that exact moment of observation never be "complete". This duality is the demonstration of its unpredictable nature and its transition from one point in time to another, without knowing or fully anticipating the outcome. In the context of the Turing machine, we never know at which point in time, if at any, the program will reach a finite state. While the Turing machine can be seen as an abstract hypothetical demonstration of an undecidable system, artists like Jean Tinguely were interested in compressing the basic concept of undecidability into a more confined format. Like Turing's example, Tinguely's machines are an abstraction of all these concepts. He wanted to produce art that would at no point be conserved in a museum, because he considered the transition of states and the corresponding aesthetics of kinetic processes to be his point of focus:

> *I wanted something ephemeral, that would pass like a falling star and, most importantly, that would be impossible for museums to re-absorb (McNay, 2016)*

Jean Tinguely a controversial figure of his time and, most of all, an artist who started to push the boundaries of what art is by raising questions about the nature of art and its relation to the artist itself. He did that by inventing kinetic machines that he claimed would be able to produce art by themselves, or art for everyone, by everyone *(Museum Tinguely Basel, n.d.)*. In 1959, Tinguely invented a machine called the *Méta-Matic* which was a part of a series in an exhibition at Galerie Iris Clert in Paris. Its subtitle was "sculptures that paint". In essence, the machine's concept was to allow visitors to operate it by attaching a clean sheet of paper and a colored pen to the corresponding clipboard and arm. The visitor would then decide how long the machine would be turned on. The machine operated by randomly moving its arm across the paper, creating what can be considered a mechanically generative piece of art, which was then free to take home for the visitor *(Institute of Artificial Art Amsterdam, n.d.)*. Figure 31 shows a picture of this machine:

Figure 31, Meta-Matics by Tinguely

(*Méta-Matic No. 10*, 2024)

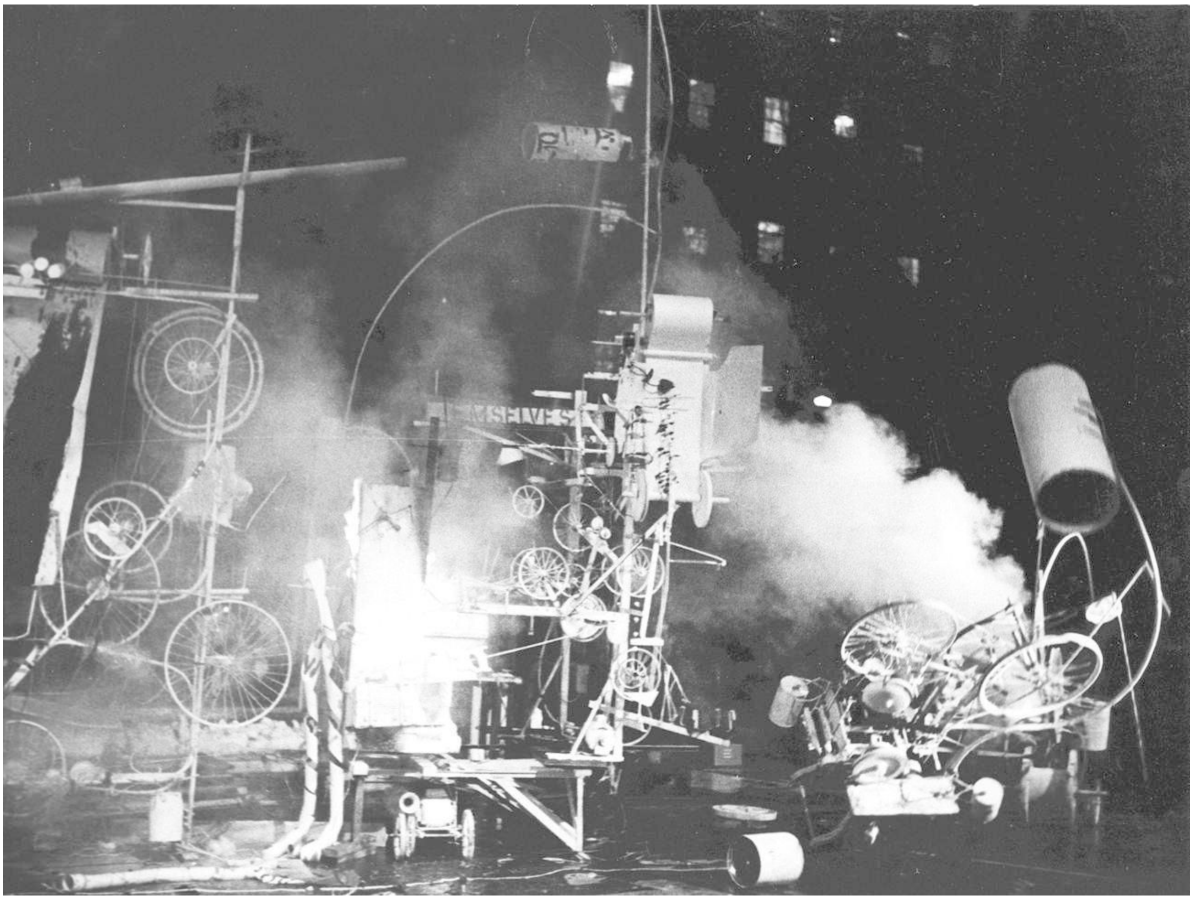
This machine underwent several iterations, and in the same year, Tinguely was thrown out from the first Paris Biennale because people were severely irritated at his interpretation of what art could be.

> *People were furious (...) [n]o one knew exactly where the art was to be found. They asked me, is the art in the apparatus, or is it in the result? I said: 'It is in between.' (McNay, 2016)*

The "in between" was the process Tinguely was most interested in. He was not concerned with the outcome, nor did he want his machines to produce finite art. Moreover, his machines were not the artworks per se, it was purely the process of kinetic movement paired with the inaccessibility of the human mind to comprehend the undecidable outcome of the "in between", the operational process. Much like the "in between process" of a Turing Machine, it is not the outcome itself, but the uncertainty of its outcome that makes it fascinating. This correlates very much with how in art very often it is the concept, the idea that generates context to the outcome and lends meaning to artistic practice. We often operate on the fringe of technological processes that are considered a stable and reliable part of our everyday lives, while our technological future and the machines we build are hardly ever anything else but temporary. It is this relationship that Tinguely also articulated through his fragile designs. They, like us, were never meant to last. And if so, then only by pure chance, by pure coincidence.

Jean Tinguely called his machines *méta-mécaniques*. The Greek word "meta" meaning "self-referencing". He disconnected the machine as an utilitarian object from its intended purpose, and gave it an existential bias in the modern age. A machine that purely exists for aesthetic reasons, a machine that interacts with humans solely for the purpose of the interaction. He built many different kinds of these machines, all with their very own "meta level". Some of them were even built to self-destruct *(McNay, 2016)*. In 1960, Jean Tinguely created what was probably the most famous artwork of his whole collection, named *Homage to New York*. It was set up as an art performance event and took place in the sculpture garden of New York's Museum of Modern Arts. Participating in the event was one of Tinguely's enormous machines which many described as resembling a mechanical Frankenstein, composed of bicycle tires, gears and electronics and a plethora of other motors, discarded machine parts and mechanics. Two other artists, Billy Klüver and Robert Rauschenberg, were also involved in designing parts of the mechanized performance by constructing auxiliary parts that shot out money into the crowd. When they started the machines, it took the whole parade of a frenzied mechanized circus about 27 minutes before breaking down and imploding into a roaring inferno of smoke and fire. Tinguely had intended it that way. After the inferno sizzled for some time, he allowed the crowd to salvage and gather what they could while the rest was eventually put out and discarded by the fire brigade *(Experiments in Art and Technology, n.d.)*.

Figure 32, "Homage to New York" by Tinguely



(*Homage to New York*, 2024)

Tinguely loved the idea of countering any attempts to contextualize his contraptions outside of their intended moment. They were deformed and often hideous caricatures of the clean and purposeful machines that dominated the technological evolution of the 20th century. They were a representation of his understanding of art: "Art is the distortion of an unendurable reality ... Art is correction, modification of a situation" *(Barcio, 2016)*.

It is almost impossible to do justice to his wonderful machines with mere words. Also pictures do not suffice due to their inherent undecidable, unpredictable and chaotic nature. Still, when looking at these phantasms of constructs, it allows one's imagination to run loose about how they might have moved, sounded, operated or imploded. No matter their meta purpose, it is only in that precise moment of "in between" being switched on and off that their purpose was fulfilled.

It is also important to note that Jean Tinguely was part of the *Nouveau réalisme* movement founded by Yves Klein in 1960. Tinguely's name can be found in their manifesto *(Tate, n.d.-d)*. *Nouveau réalisme* was dedicated to exploring the boundaries of art and their members were on a quest of finding new ways of perceiving the real. The second half of the 20th century was a time often dominated by drastic changes. It was the transition into the age of technology and the age of the renaissance of the machines after the industrialization. Global technical advances and the geopolitical situation turned into both a nightmare, garnished with the constant threat of total nuclear annihilation, and hope, much like in *Star Trek*, that we as a species could some day "make it": to have a prosperous future without the sacrifices of the past. It was a time still shaken by the aftermath of the horrors of World War II, and the world had to be rebuilt and reimagined by artists like Jean Tinguely and dreamed by his machines.

To me, Tinguely's machines are a manifest of themselves, breaking the constraints of what art is allowed to be. It is a playful approach to the ideas of the unpredictable and chaotic nature of what reality really is. It was a heartfelt goodbye to the idea that art is anything else than a representation of a moment that has no past and exists also within itself without having a predictable future. It also questioned the roles and relations between technology and the artist as a subject. Who is the artist if a machine executes what we ask it to? Can a machine be an artist too? These questions also get a lot of momentum in recent days since the advent of artificial intelligence. Perhaps, we stand at the brink of what artists like Jean Tinguely tried to warn us about. It is as if he already predicted that we should be weary of our relationship with technology in the future, because it is easy to get distracted and subsequently replaced without meaning. We have to be careful not to become "meta machines" ourselves. I want to close with a quote from Jean Tinguely himself:

> *To play is art – consequently I play. I play enraged. (McNay, 2016)*

Figure 33, Meta Machines by Jean Tinguely

(*Gismo*, 2017)

(*Meta Machine (unknown)*, 2024)

(*Méta-Harmonie II*, 2024)

(*Méta-Maxi-Maxi-Utopia*, 2015)

(*Heureka*, 2024)

(*Dissecting Machine*, 2018)

Artpocalypse

In the more recent past we have seen the dawning of a new age of digital art. It is a practice that has been shrouded in mystery for years and seems to have sprung into existence out of nowhere: AI-generated art. What first started to receive arguably minor public attention, as a sort of weird experiment started by the Google AI company, has now found its way into a broader audience.

In 2015, Google started to experiment with its AI which was responsible for identifying images fed to its neural network through the internet. Back then, it used a method called "backpropagation" which used 10 - 30 layers of artificial neurons which where trained by feeding its algorithm millions of images in order to slowly adjust the network parameters for finding the correct answers, i. e. identifying what is shown in every single image *(Google Research, n.d.)*. Engineers then reversed the process and wondered what would happen if they turned the tables by feeding the machine with attributes instead of images in order to create an image rather than identify the correct parameters of an image. The results where quite strange in the beginning, but as the capacities of their neural network advanced, the results soon showed mesmerizing dreamscapes *(Google Research, n.d.)*. Many of them looked like they had originated from a lucid dreamscape from a collective, interwoven subconscious of man and machine. It somehow created a mirror world of our own digitalized cultural efforts and left some of us terrified with what was about to emerge from the depths of this kind of technology and its descendants.

See figure 34 on the next page for some examples of Deep Dream:

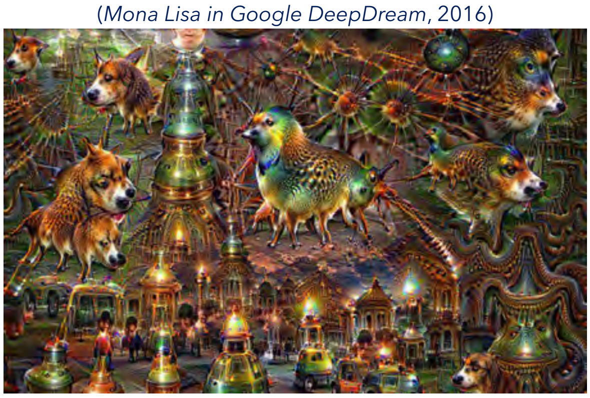
Figure 34, Google AI Deep Dream Images

(*Vincent Van Gogh in Google DeepDream*, 2020)

(*Random Image in Google DeepDream*, 2024)

(*Starry Night in Google DeepDream*, 2016)

(*Mona Lisa in Google DeepDream*, 2016)



(*Creepy Dream in Google DeepDream*, 2015)

While at first it seemed like an odd anomaly in the context of digital novelties we have accustomed ourselves to over the past decades, a lot of people realized that this was only the beginning of a revolution in how we can interact artistically and intellectually with these advanced technologies.

15 years earlier, Google founder Larry Page said in an interview:

> *Artificial intelligence would be the ultimate version of Google. So we have the ultimate search engine that would understand everything on the Web. It would understand exactly what you wanted, and it would give you the right thing. (Academy of Achievement, 2016)*

And indeed, what they created with the Google AI algorithm was a system that was capable of identifying and delivering on just "exactly what you wanted". It was created as a system that decides if the picture it was fed was in fact a dog or a cat, or anything else one could imagine. Soon, we became accustomed to the fact that no matter what, we could just search for it on Google and get the answer - and it became really good at that. Thus, I would argue that it can be defined as a decidable system. Once the data has gone through all the possible peers inside its own neural network, the machine "decides" what the "correct" output is.

But when the process is reversed, it suddenly turns into an undecidable system where the output becomes a machine-interpreted approximation of what the defined parameters might be, a dreamscape of a very potent non-sentient entity. It possesses many of the inherent attributes of an undecidable system, since the output changes every time the algorithm "dreams", even when given the same input parameters each time; and it also learns and expands with each consecutive input calculation, making it impossible to predict its future outcome.

"Do androids dream of electric sheep?", famous science fiction author Philip K. Dick asks with the title of his 1968 novel. The answer might be yes, if given the right framework of input parameters. While his question, as a visionary of his time, most certainly aimed more towards the philosophical issue the ultimate definition of life itself and the possible answers' implications, one might also use this prompt as an opportunity to ask: What is imagination? Or, to rephrase it in a way fit for this context: Can an AI that produces images be said to have a certain degree of inherent creativity? Can a machine also "imagine" things and thus be creative?

There is an interesting fact about these kinds of systems that is described as a "black box" mystery *(Levy, 2024)*. If the complexity of artificial neurons has reached a certain threshold, it becomes very difficult or almost impossible to distinguish which parts or patterns of neurons are responsible for a specific decision (i. e. a specific outcome to a task) made by the network itself. The possibilities simply become too vast to apply sufficient scientific tools for a detailed analysis of the networks' "inner workings" when searching for a specific property or as Henry Lin, a physicist at Harvard University puts it: "The big mystery behind neural networks is why they work so well" *(Ghose, 2016)*.

On August 30, 2022, a tweet shook up the art world and was considered by many as a step over the red line. It generated an immense outrage and was the origin for many of the discussions that followed concerning the basic values and consent of the artistic community in general.

Figure 35 on the next page shows a screenshot of the tweet:

Figure 35, "Yeah that's pretty fucking shitty." Tweet

(*Yeah That's Pretty Fucking Shitty*, 2022)

For the first time in history, a machine generated artwork had won the first prize of a fine art competition in the category for digital art. The artwork is called *Théâtre D'opéra Spatial* and was generated using Midjourney, a discord based AI bot that takes prompts and turns them into his interpretation of whatever the user inputs into its algorithm.

Figure 36 on the next page shows the actual picture:

Figure 36, "Théâtre D'opéra Spatial" by Midjourney AI

(*Théâtre D'opéra Spatial*, 2022)

Jason Allon, the creator of the artwork, ran his prompts through the AI hundreds of times in order to refine and reiterate on his input prompts and finally choose three images which he thought best resembled his vision for his artwork *(Kuta, 2022)*.

It was a profound development: An undecidable system was able to convince a jury that what had been produced through possibly endless iterations by an algorithm can at one point be considered art. An artistic machine let loose on its own terms, permanently fed by the curiosity of human beings in order to stretch out the possibilities of its creative potential by exploring the impact of every single parameter fed into the system. Here, the artist has become the computer and the computer has become the artist. And it feels like we are just touching the tip of the iceberg, with many more image creating AI models seemingly on the rise, and ever more constantly entering the ring.

Of course, this prompted a cry of outrage in the artistic community, since many consider true art as something that can only originate out of the ideas, skills, and imagination of sentient beings who use technology only as a tool to bring their works into existence. This is interesting, because it begs the question where to draw the line between using technology as a tool, since these AIs do not create images "on their own" without human interaction. On the other hand, all the attributes of the final artwork that count towards an artistic classification are created by the machine without any actual doing of its handler. It creates an answer to an artistic approach never attempted by the artist per se. It manifests art through input and ideas not thought up by itself, yet still imagines every single piece of the puzzle through the inner works of its vast neural network.

One could argue that what makes these tools so powerful is their intrinsic undecidability in how they interpret our own ideas. In my view, it can be reasonably concluded that art can originate from undecidable systems; moreover, in this context, art can become something inherently undecided.

Nagarjuna

Looking into the abyss

> *There was a moment when the grammar of the world seemed clear: at the root of the variegated forms of reality, just particles of matter guided by a few forces.*
>
> *Humankind could think that it had raised the Veil of Maya, seen the basis of the real. It didn't last. Many facts did not fit. Until, in the summer of 1925, a twenty-three-year-old German spent days of anxious solitude on a windswept island in the North Sea: Helgoland - in English also Heligoland - the Sacred Island. (Rovelli, 2021, p. xiv)*

As described in Carlo Rovelli's book *Helgoland*, it was on this island where the young physicist Werner Heisenberg went in 1925 to isolate and immerse himself into complete solitude. Alone in this landscape of extremes, he sought to free his mind of all preconceptions and distractions in order to confront one of the most profound problems in the history of our human minds: Finding the key that unlocks the path to the core of what separates ourselves from what we call reality, from everything that surrounds us. Understanding what to conceive something as "real" even means. Going on a journey through the vast microverse filled with the building blocks of everything at the very basis of our perception and to seek out the mysteries reality has been hiding for so long.

For a very long time through history, we eased our philosophical and artistic hunger for the objective truth about the universe in a very materialistic way. We came to understand that the things that surround us are the consequence of cause, materializing in its final form for the moment we call the present. A stone, for example, is the manifestation of its geological past, it is a representation of an objective existence of things prior to us classifying and categorizing it as a stone and thus applying attributes to it as an object. It is intuition that weaves the logic for our own constructions and the way we force our minds to bring order to the chaos. We think of ourselves as entities immersed in a cosmos of other entities that exist by themselves, on their own, so to say. Existence itself always qualified as a principle transcending our need to acknowledge something "is" existing. In other words: The stone exists even if nobody would ever know it was there. Things exist independently of each other, as their own entity, throughout the universe. We, as humans, are just wanderers that explore an objective reality in which we exist and which would prevail even if we ourselves would cease to exist along that trail. This is the point in time where Heisenberg, along with some other brilliant minds of his era, went out and found light at the end of the deterministic tunnel. This is where reality as we knew it collapsed into a reality of relations and probability rather than objects or entities. It was the birth of quantum theory.

> *The abyss of what we do not know is always magnetic and vertiginous. But to take quantum mechanics seriously, reflecting on its implications, is an almost psychedelic experience: it asks us to renounce, in one way or another, something that we cherished as solid and untouchable in our understanding of the world. We are asked to accept that reality may be profoundly other than we had imagined: to look into the abyss, without fear of sinking into the unfathomable. (Rovelli, 2021, p. xviii)*

Like in previous chapters, I want to to focus more on the implications of these extraordinary events and conclusions in the context of quantum mechanics, rather than try to dissect and translate the ultra-complex mathematical and physical axioms from which it derives. As a matter of fact, quantum theory quite naturally sits at the fringe of many different disciplines. While deeply rooted in the analytical apparatus of science, it is at the same time something so philosophically mysterious and wonderful that even physicists like the recent Nobel laureate Anton Zeilinger, alongside many of the greatest minds presently studying the subject, have to confess in some way or the other that they are completely baffled by what they have found. To illustrate, consider the following selection of quotes from famous scientists about this *(Ferrie, 2023):*

> *'Quantum mechanics makes absolutely no sense.' – Roger Penrose*

'If it is correct, it signifies the end of physics as a science.' – Albert Einstein

'If you are not completely confused by quantum mechanics, you do not understand it.' – John Wheeler

'I do not like it, and I am sorry I ever had anything to do with it.' – Erwin Schrodinger (sic!)

What they are implying is that we can observe its effects, make the right predictions, and make effective use of the theory while at the same time remaining completely perplexed by the conclusions we have to draw because of the fact that it works in a way it should not, according to how we thought "things" generally are. It absurdly tells a tale of a reality that exists because it does not exist, or as Časlav Brukner, a Serbian-Austrian expert on quantum physics, puts it: "Can this be believed? It's as if reality ... didn't exist..." *(Wright, 2021)*.

What does that even mean? Up until the 20th century, the prevailing view was as follows: In the beginning of everything, there was a lot of matter which was comprised of different kinds of particles floating around the universe. This entropic mixture was combined with a great amount of energy which resulted in more complex behaviors of these clusters of matter. All of it was made out of extremely small buildings blocks which we later came to name atoms. Eventually, those atoms formed more complex structures and got mixed together into all kinds of different elements. These elements, over vast amounts of time, started to form objects through the sheer force and energy of gravity and heat. Soon, we had suns, planets, fluids, metals, gases, and all other kinds of cosmic entities. Eventually, there were animals and humans, who after some time evolved into somewhat sentient creatures who were able to figure out all the above and felt very proud of themselves in doing so. Cause and effect was declared the primary phenomenon behind everything *(Pultarova, 2022)*. We applauded our own mind that it was able to comprehend what reality truly is. It is made out of materialistic objects that surround us and that we can study in order to understand and harness its properties. But, ever the curious creature, we were not fully content with what we had discovered so far. We went and dug deeper. The goal was to further dissect and understand the building blocks of literally everything in existence. We wanted to know what the atom was made of. And at that exact moment, we accidentally opened Pandora’s box.

> *The most merciful thing in the world, I think, is the inability of the human mind to correlate all its contents. We live on a placid island of ignorance in the midst of black seas of infinity, and it was not meant that we should voyage far. The sciences, each straining in its own direction, have hitherto harmed us little; but some day the piecing together of dissociated knowledge will open up such terrifying vistas of reality, and of our frightful position therein, that we shall either go mad from the revelation or flee from the deadly light into the peace and safety of a new dark age. (Lovecraft, 2021, p. 5)*

So how does an atom work? How do the electrons inside it behave? How do they actually move? We assumed that, with enough scientific precision and effort, we should be able to predict the orbit of even the smallest particles, such as an electron. Since it is still some kind of matter, as we reasoned, we should be able to observe its position and speed relative to the nucleus of the atom itself *(Pultarova, 2022)*. Much like we could measure the speed and trajectory of any other object, like that of a thrown stone. The point I am trying to make is that if reality exists as a collection of independent entities, it must be unbothered by us looking to measure its properties. Thus, at any given point in time, if we look close enough, we can take a picture of what state something is in along its deterministic path. It turns out that reality is having none of that. That would be far too easy.

As of writing this paper, only two years ago, Alain Aspect, John F. Klausner, and Anton Zeilinger were awarded the Nobel prize for physics "... for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science" *("Pioneering Quantum Information Science", 2022)*. This sentence is not expected to be comprehensible for laypersons like me, but it can roughly be translated as follows: "The universe is not locally real" *(Garisto, 2024)*. Other than the hilariously shocking conclusion of the bare statement alone, the two important words in this sentence are "locally" and "real". So in this context, they mean the following *(Garisto, 2024)*:

Locality: As we learned before, the world we thought we knew was a deterministic one. Cause and effect are at the root of everything while the maximum speed for any object or particle in the universe is the speed of light.Thus, any cause and effect does not move faster than the speed of light. This is also the reason that when looking through a telescope, we only ever see the *past* of the object we are observing. A star that shines bright in the present night sky may have long ceased to exist. If, for example, a planet or star would be one light year away from us, we could still observe it for exactly one year after its demise. That is because the visual information of its destruction is nothing more than light reaching our eye through the telescope which, at the speed of light, needs exactly one year to traverse that distance.

Realness: Until the 20th century, the view of the materialists like Albert Einstein was that the universe is real. This means that every particle, down to the smallest building blocks of reality, have definite properties. These properties are inherent to their existence, regardless if anybody was actively trying to measure those attributes. In a nutshell, this is the more intuitive question to the popular question "If a tree falls in the woods, does it make a sound if nobody is around to listen?"

Even though the answer to this questions might seem obvious at first, it is far from being decided up until today. The movement opposing the views of these materialists back in the 20th century was called the "anti-realists".

At this point, I want to emphasize that this simple question is actually also inherently undecidable, because it is neither scientifically verifiable nor falsifiable *(Baggott, 2011)*. Most of the time, it is just our very handy intuition that often shields us from making the wrong predictions. Thus, we rightfully assume that things generally happen even if nobody is there to witness them. However, people like Niels Bohr, Werner Heisenberg, Ernst Schrödinger, Arthur Schopenhauer, and many others were convinced that this was, in fact, probably not the case. They suspected reality to be far more complex and strange than it seems to the naked mind. They had also come to a point as physicists where their work took a spooky turn and slowly but gradually arrived at a conclusion fundamentally different from the one of the materialists: The properties of any particle (and thus basically everything that exists) are inherently undecided until someone or rather something makes the effort to measure them *(Lewton, 2023)*.

One famous example, that is a bit simplistic in its grade of detail and serves more like an analogy rather than a real experiment, is *Schrödinger's cat*. It is essentially a more confined version of the falling tree in the woods *(Matthias, 2023)*: Imagine a cat in a box you cannot see into. That box also contains a capsule of sleeping gas that can be randomly triggered. In the original version, the capsule contains a lethal toxin, because Schrödinger was a savage - I prefer Carlo Rovelli's variation *(Rovelli, 2021, p. 52)*. You never know if or when the sleeping gas is triggered and makes the cat fall asleep. That means, from your perspective as an observer, as long as you do not open the box, the cat is simultaneously asleep and awake. It is in a so called superposition of both states relative to your own point of view *(Matthias, 2023)*. Its true state of existence in relation to being awake or asleep is undecided. So while the box remains closed, Schrödinger's cat is an undecidable system. As soon as the box is opened, the riddle is solved: The cat is either asleep or awake, since it obviously cannot be both at the same time.

An intuitive reaction to this example would be to say, "What nonsense! The cat was always either asleep or awake in the first place, even if nobody knew for sure". It turns out that this is not the case. Indeed, the cat is, relative to the observer, both awake and asleep at the same time on an atomic level. And we can prove it. Quantum theory is one of the most tested scientific theories of the past 100 years:

> *[It] has clarified the foundations of chemistry, the functioning of atoms, of solids, of plasmas, of the color of the sky, the dynamics of the stars, the origins of galaxies . . . a thousand aspects of the world. It is at the basis of the latest technologies: from computers to nuclear power. Engineers, astrophysicists, cosmologists, chemists and biologists all use it daily; the rudiments of the theory are included in high school curricula. It has never been wrong. It is the beating heart of today's science. Yet it remains profoundly mysterious, subtly disturbing. (Rovelli, 2021, pp. xiv-xv)*

So what it predicts about the cat in the box is also valid in the microverse, also known as the realm of particles. If we consider electrons, for example, we always imagined them as some kind of structural objects which would traverse through a specific orbit around their respective nucleus. This means that at any given time, an electron has at least a definite position and speed. However, this is not the case. Without diving too far into the physical specifics, quantum theory and its respective champions like Niels Bohr, Werner Heisenberg, Richard Feynman, and thousands of others, have since proven that really anything that "exists" does not exist until it is measured by something else *(Ananthaswamy, 2024)*. The electron, for example, is in the previously mentioned state of "superposition" which means that, relative to the atom, it exists in every possible configuration at once while also acting like a wave rather than a particle. And in the exact moment we look at it, that wave of probabilities collapses to a definite particle we can observe.

As strange as it sounds, but nature itself somehow appears to "care" if it is observed by itself (for example, by human beings) in relation to every object that is interacting with another observable attribute in any way with anything else. And if there is no observer, reality does not exist at all or, to be more exact, is at least in an undecided state *(Ananthaswamy, 2024)*. It also means that because there are no given or predefined attributes of any object, before reality manifests, it only exists in a wave of probabilities. Schrödinger called this the psi wave. Because I suspect the complexity of the implications to be a bit overwhelming at this point, I will try to clarify what the study of quantum mechanics has proven up until now:

- The microscopic world behaves vastly different from the macroscopic world (essentially Einsteins theory of relativity vs. quantum theory)

- In the microverse, particles behave as if they themselves are waves and particles at the same time containing all possible options of their measurable attributes condensed into something called a "superposition"

- Only as soon as we measure or observe any given attribute (e. g., the position of an electron) the wave function of probability (psi) collapses and the electron will "decide" where exactly it physically manifests into a particle at the exact moment of measurement

- If nothing is "observing" the particle, it does not manifest into reality and thus, because everything is made out of atoms, nothing exists a priori.
- The existence of objects is related to the existence of other objects

God does not play dice

"God does not play dice" might be one of the most famous and most misunderstood quotes that made its way out of its scientific origin and is broadly used as a synonym for "There is no such thing as pure chance" *(Baggott, 2024)*. It is often used metaphysically while at the same time legitimizing its apparent literal expression by the notion that the great Albert Einstein himself was its creator. At first glance, it implies two things: That there is something like a god who is the orchestrator of "existence" or "reality" and that this god at the same time does not permit anything happening by mere chance. As previously mentioned, Einstein used the term "god" as a synonym for the totality of existence or the rational* character of reality (*i. e. to some degree accessible for the human mind). As stated before, the "not playing dice" aspect is tied to a notion of reality that is purely deterministic *(Baggott, 2024)*. The whole quote originates from a letter Albert Einstein wrote to fellow physicist Max Born because of his dissatisfaction with the back then pressing revelation which hinted at the preeminence of probabilities and chance introduced to the elements of reality by breakthroughs in the newly established fields of quantum mechanics. The original quote from this conversation is "I, at any rate, am convinced that [God] is not playing at dice." *(Einstein et al., 1971, p. 91)*

The concept that cause and effect cease to exist at the quantum level was deeply shocking from both a philosophical and scientific perspective, as it meant the end of all hope of being able to decipher all there is by simply refining the methods until all of the deterministic causes and effects of any system could be measured. Instead, it introduced us to the fact that the very core of everything that is and will ever be is more probably an undecidable system, a state of all possible probabilities which, at the glance of an eye, will manifest its final form into one possible outcome above all other by mere chance. The final state of anything on a quantum level at any given moment in time when being "observed" is, in fact, truly random.

Moving past the quantum world, the concept of randomness has also always been a faithful companion of a lot of creative efforts in art and design. Especially with the rise of digital tools as means of expression, we as artists often use some instance of random occurrences as catalysts for our own artistic endeavors, even more so in the field of media art. It is such a powerful tool that one might find it challenging to consume any sophisticated interactive media art installation that has not at least a single, seemingly random aspect that influences the recipient's experience. As previously explored in this paper, it is also one of the driving forces of evolving systems in generative and algorithmic computer art. The concept of randomness itself is almost as old as mankind itself, and one of its most prominent manifestations as a fixed point in culture that needs no further introduction comes in a cubic form: a simple, six-sided dice. Another obvious example which needs no further elaboration is, of course, the iconic coin flip.

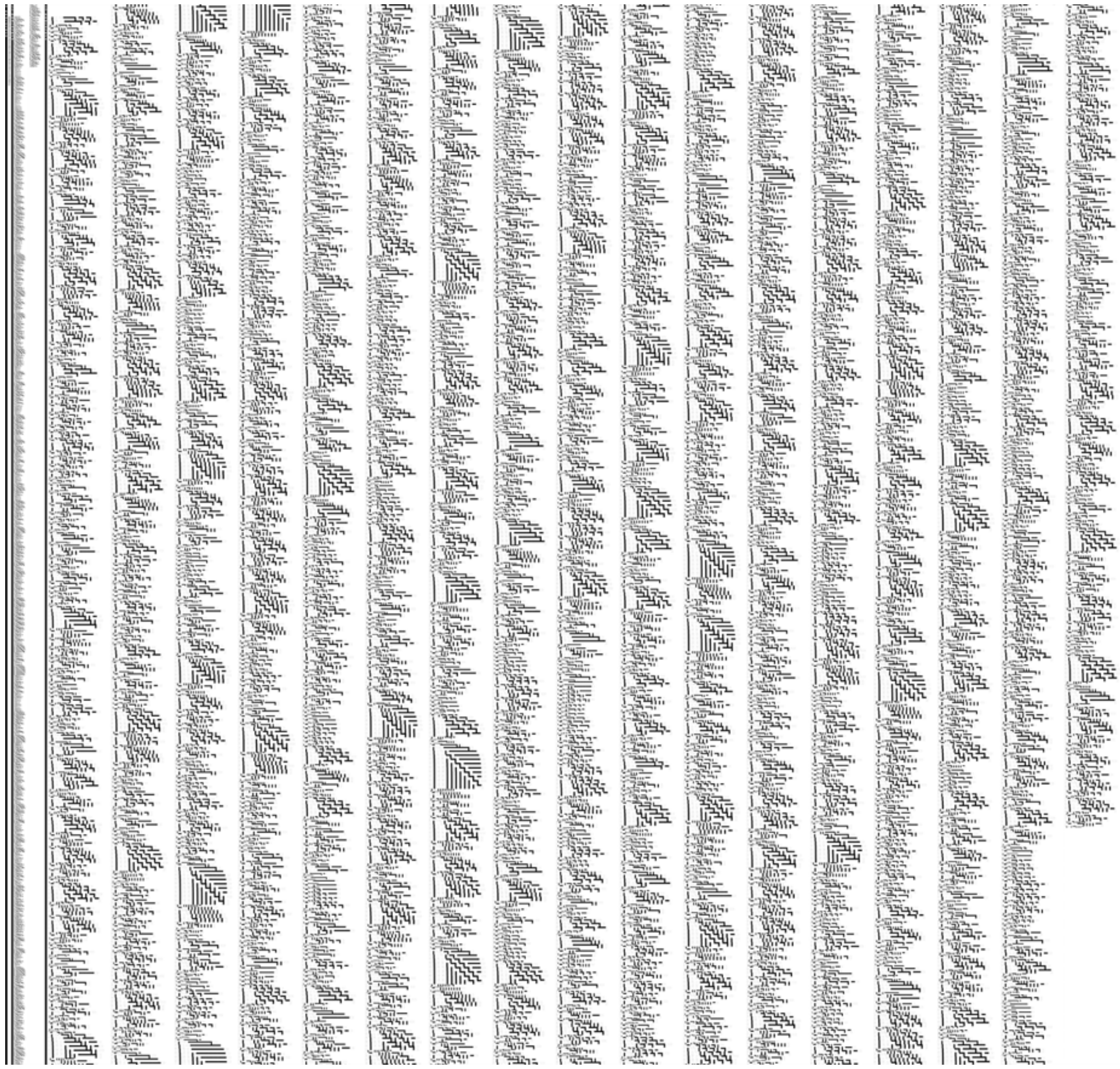
I would even argue that without rudimentary elements of chance, there would be no such thing as "homo ludens". But what exactly is randomness? The concept the term describes is actually much more complex than what first meets the eye, even outside of quantum mechanics. The artist and designer Carl Lostritto opened his article about "The value of randomness in art and design" as follows:

> *Ask a designer or artist if any aspect of their process is random. The answer will likely reveal a complex relationship between human cognition, digital media, authorship, and even conceptions of reality and the divine. For those of us who work in computational media to make art, the question can be even more focused: When and why do you use a "random()" function when you write code? (Lostritto, 2015)*

By concept, the intentional randomization of an artwork's different attributes is often a hidden layer behind the visible outcome. However, it can also serve as an explicitly presented, major feature. There is art that uses randomness as a factor for aesthetics, sound and visuals, and then there is art that focuses on the concept of randomness itself. In the following, I want to briefly touch on one example of visualizing randomness as a concept. Note that the following artwork by Rami Hammour is able to combine two aspects: It represents a momentous visualization of what randomness can look like if contextualized by mapping random values to generative forms or patterns, but at the same time the algorithms used to determine those numbers and patterns do not need to be truly random at all. Any structure chaotic enough for our brains to be considered random will suffice, while the real process behind the structure can remain completely reproducible with a fixed and predictable system. I will elaborate more on why that matters later.

Figure 37 on the next page is taken from Hammour's artwork *A Text of Random Meanings* and is a representation of one of the standard methods used in computer science to produce seemingly random values. This method is broadly known as the "shift-register and tap"-method *(Dunn, 2011)*.

Figure 37, "A Text of Random Meanings" by Hammour



(*A Text of Random Meanings*, 2023)

For the sake of completeness, here is a short description of the underlying mathematical theory from Burton Rosenberg, professor of computer science at the University of Miami:

> *The shift-register pseudorandom number generators are calculating the successive powers of an unknown x in a finite number field called the Galois (gall-wah) of order 2 to the n. To understand, first consider the simplist finite number field. Take a prime p and consider the integers mod p. They are 0, 1, 2, etc. up to p-1, with addition and multiplication mod p. Since p is prime (and only if p is prime!)*

> *with the exception of 0, for each x there is a y such that x*y = 1 mod p. That is, each number except zero has an inverse mod p without resorting to the invention of fractions. (Rosenberg, 1997)*

While for the context of this paper it is not necessary to understand any of these mathematical expressions and theories, this quote introduces the second of two interpretation or subcategories of the term "randomness" that we have encountered so far. The first one is the previously mentioned "truly random", the second one is "pseudorandom", as in the above quote. Also, no matter how complex the mathematical explanation of how to reach something considered random might seem, every "process" of "creating" something random begs the question if something where we know exactly how it can be reproduced and "why" its outcome "seems" random can actually be random at all. Especially since our modern computers are, in essence, entirely predictable. At this point, let us take a step back: What actually is randomness? Apart from the intuitive perception of what it means, what actually is its definition? The Merriam-Webster dictionary offers the following:

> *Randomness : the quality or state of being or seeming random (as in lacking or seeming to lack a definite plan, purpose, or pattern) (Merriam-Webster, n. d.)*

There is something very interesting about this definition. Not only is it defined as something that "is", but at the same time also as something that "seems". If this seems strange - well, it genuinely is. What does this actually mean? As we have already learned before, it means that there are different kinds of discernible randomness which can generally be broken down into two categories: true randomness and pseudorandomness.

Notice how I use the term "true randomness" when talking about observable effects in nature i.e quantum mechanics. True randomness is what most people mean when they use the the term "random", because we are often tricked into experiencing something as subjectively "truly" random (i. e. the unpredictability of a changing state or system) while it actually is not. However, most of the time, when we use the term randomness or a random system like a dice or a coin flip, it actually turns out to be pseudorandomness or "not being objectively random at all". To clarify, true randomness is "when a phenomenon is intrinsically random and not dependent on our knowledge of the phenomenon" *(Prince, 2022)*.

Just consider a simple throw of a dice during a board game. The only reason we perceive it as random is because prior to the throw, we have insufficient data at our disposal to make any meaningful predictions about the outcome of the throw. We all know the somewhat playful, sometimes slightly desperate attempts at "cheating" in obvious ways by trying to not throw the dice too hard and trying to make it land on the side with our desired number on it. Of course, this is hardly crowned by success. However, just imagine having all the tools and knowledge at your disposal to calculate the necessary forces and angles to achieve your desired result at the throw of a dice. Theoretically, this would actually be possible in a confined environment with sufficient hardware and calculations. One could simply build a machine that can perfectly throw a dice every time, which makes the outcome 100 % predictable. In fact, a machine that can do the same for coin flipping has already been built successfully at Harvard University under supervision of Mary V. Sunseri, a professor for statistics and mathematics *(Diaconis et al., 2006) -* and a coin flip is essentially just simplified version of the dice throw. My point is that everything that would be theoretically predictable with sufficient data and analysis can never be truly random in the pure sense of the word, although it is random enough that it will suffice for playing a board game or any other arbitrary system with chance as an integral element. We can state that if the outcome of something is defined by any set of initial conditions, it must be considered "not random" or pseudorandom *(Diaconis, 2007)*.

The same applies for the digital realms of media art, where most of the time the triggering of anything random happens through algorithms that only pretend to be random. No computer can create any kind of true randomness intrinsically:

> *'One thing that traditional computer systems aren't good at is coin flipping,' says Steve Ward, Professor of Computer Science and Engineering at MIT's Computer Science and Artificial Intelligence Laboratory. 'They're deterministic, which means that if you ask the same question you'll get the same answer every time. In fact, such ma-*

chines are specifically and carefully programmed to eliminate randomness in results. They do this by following rules and relying on algorithms when they compute.' (Rubin, 2011)

Still, there is a way to increase the level of randomness a machine or a computer can produce by introducing external, unpredictable factors that are considered truly random as a reference point for the machine's algorithm, for example thermal or atmospheric noise. At this point, it is worth noting that one could still argue that even these seemingly unpredictable factors of nature are yet to be proven fully random as well. Just as with our coin flip or throw of a dice, it could be theoretically possible to predict any kind of external factor as well, even if the complexity would be close to almost impossible. This in turn raises the question whether any circumstance of the natural world that seems truly random is indeed truly random or just indistinguishable from true randomness due to its extreme entropy.

So far, we can conclude that trying to solve the equations which would be able to determine true randomness outside of quantum mechanics is very much an undecidable problem. This leads to an interesting paradox: If a system would be truly random, the possible outcomes of that system would also again make it an undecidable system by definition. So what, then, actually *is* truly random?

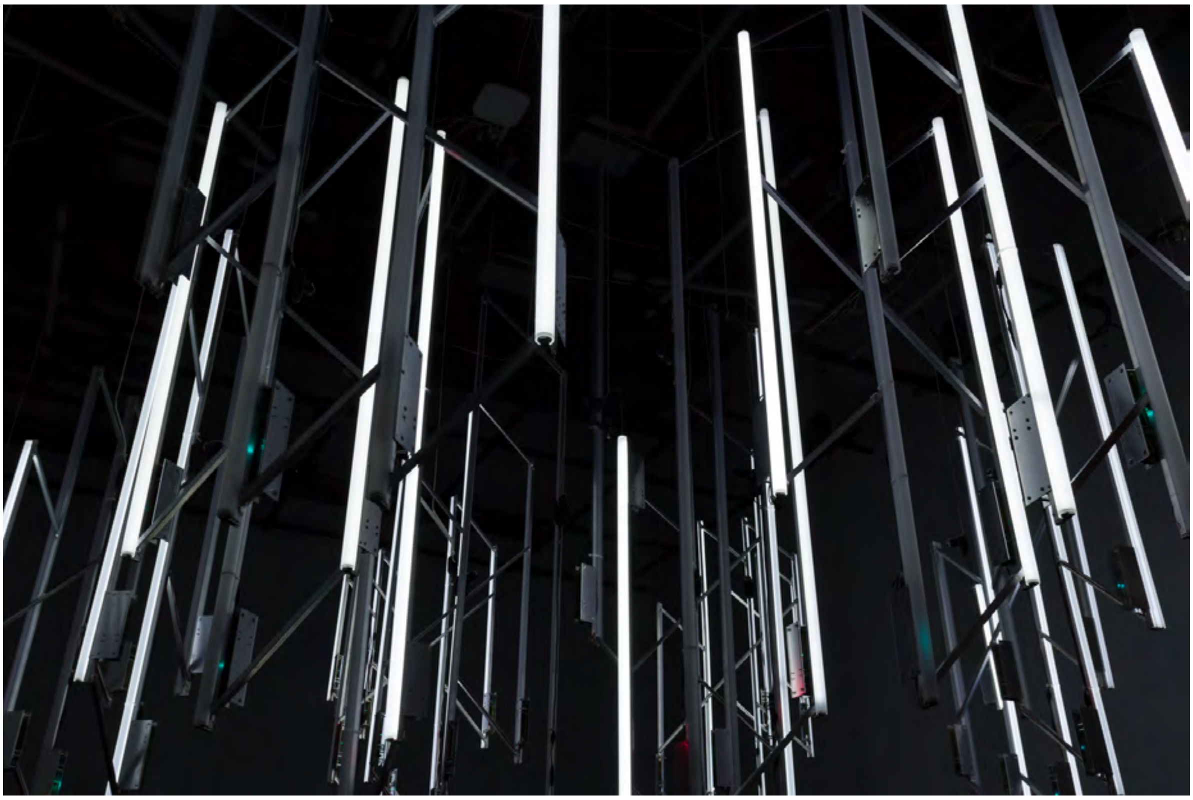
The artist Phillip David Sterns created an artwork called *A Chandelier for one of many possible Ends*. It is an installation which consists of 92 light elements, each of which is connected to its own Geiger counter. This makes every individual unit sensitive for ambient radioactivity. Every time a radioactive event is detected by the sensors, the corresponding element emits a brief flash of light along with an audible click *(Stearns, n.d.)*. Take a look at figure 38 on the next page for a better understanding of the assembly:

Figure 38, "A Chandelier" by Phillip David Sterns

(*A Chandelier*, 2014)

(*A Chandelier*, 2014b)

Figure 38, "A Chandelier" by Phillip David Sterns



(*A Chandelier*, 2014c)

As the artist himself states, this artwork was heavily influenced by the Fukushima Daiichi nuclear disaster on March 11, 2011, when the plant was flooded by a tsunami triggered by a magnitude 9.0 earthquake. The resulting meltdown in three of the plant's six nuclear reactors released substantial amounts of radioactive material into the environment. *(Stearns, n.d.)*

It is a sculpture that should warn us about ourselves as a species and the power we harness in the modern age. As history has shown us, we are probably not yet ready for our own most intricate inventions and technical advances, as we still have not learned to trust each other enough to avoid mutual destruction on a multitude of levels. It is a spooky apparatus that also has been lent its title from the world of the macabre. Invisible radiation is made into an experience that succeeds in startling the unsuspecting visitor by demonstrating that that which is hidden from our sights is still real and that we are never truly safe. It succeeds in reminding us that radioactive events are a very real and in fact also a frequent occurrence, even without disaster. It is one of many dials controlled my mankind, which could possibly escalate into invisible and inevitable death for almost all of our planet's species -

with just the least bit of carelessness, the smallest mistake. And while the inspiration of this particular piece lies in the past, it has never been more relevant than today.

Apart from its eerie message, this artwork uses what can be considered true randomness, as its Geiger counters measure radioactive decay. Radioactive decay is considered a "stochastic" process on an atomic level which means that, according to quantum mechanics, it is impossible to predict the exact moment of time when a single atom will decay, no matter how long its lifespan has already been *(Thirumurugan, 2020)*. The way we define a probable lifespan of any sum of identical atoms is called "half-life", which describes the time required for a quantity of a substance to decay to half of its initial value *(The Editors of Encyclopaedia Britannica, 1998b)*. While highly unlikely, it is also theoretically possible for any individual atom to completely ignore any predictions of its decay and not disintegrate for as long as the universe exists. Or, on the other hand, it might decay instantaneously after its inception. This is what makes radioactive decay universally an undecidable system. Only for exactly this reason can we confidently conclude: While the world does not seem to play after deterministic rules formerly assumed true by many (including Albert Einstein), at the same time, true randomness probably only exists within the quantum states of everything that is. This means that nothing we experience in our natural world outside of quantum effects might actually be truly random, while at the same time, everything is nothing else but truly random at its very core because of quantum effects on a subatomic level. But even that theory is still up for debate, since as of yet we cannot be sure that even what we conceive as true randomness in quantum mechanics as a truly random process actually also *is* one or only *seems* to be, for now *(Aaronson, 2014)*.

As an artist, I think it is equally important to have an understanding of the underlying principles of what reality actually is or might be. As soon as we embark on our artistic endeavors, in order to reinterpret and contextualize all the different aspects we are interested in, we automatically reframe a part of reality and transform it into something new for others to experience. Not knowing about the true identity of things, even though many remain mostly highly subjective, might lead to fundamental flaws in the assumptions that preempt what we want to express with our artworks. Knowledge is the conveyor belt on which meaningful messages are transported. Art itself is nothing more than a sometimes terrifying, but beautiful message. To create art means to assume. And assumptions are, at best, the children of preliminary knowledge to some degree. But in the case of undecidable systems, this seems like an almost impossible task. Even when we ask the question if reality itself only *seems* to be or actually *is*. Or as Immanuel Kant put it in his famous *Critique of pure reason*:

> *Space represents no property at all of any things in themselves nor any relation of them to one another, i.e., no determination of them attaches to objects themselves and would remain even if one were to abstract from all subjective conditions of intuition. For neither absolute nor relative determinations can be intuited prior to the existence of the things to which they pertain, thus be intuited a priori. (Kant, 1998, p. 159)*

An artificial (r)evolution

While god might not throw dice, we as artists are certainly inclined to do so. And when talking about evolving systems in media art, one also has to take a closer look at what is called "artificial evolution". The term has originated from the respective fields of computer science and evolutionary biology. It describes a process in media art where digital organisms or structures are undergoing an evolutionary process in which they change their respective states or their audiovisual aesthetics over time, following an artificial ruleset for selective processes *(Sims, 1991)*. In media art, it is common to have some kind of visual representation of these processes which are therefore often referred to as "artificial aesthetic evolution" *(Whitelaw, 2002)*.

The journey begins with a program called *Biomorph Land* from renowned evolutionary biologist and author of *The Selfish Gene*, Richard Dawkins. While this program was mainly used in accordance with his book *The Blind Watchmaker* and is explained in broader detail in his paper *The evolution of evolvability*, its main purpose is to demonstrate the ideas of evolution as an argument against a broader variety of creationist ideas *(Dawkins, 1988)*. Concerning media art, it also still stands on its own as some kind of starting spot for what we now call artificial evolution. So what is Biomorph Land all about?

The term "biomorph" itself is a homage to the painter Desmond Morris and his surreal paintings of animal-like shapes *(Dawkins, 1988)*. See figure 39 for reference:

Figure 39, Biomorphs by Desmond Morris

(*The Arena*, 2024)

(*Totemic Decline*, 2024)

The program itself was Dawkins' attempt to generate shapes and patterns on the screen which, while simple in their geometric features, are much less the product of some kind of design from the "user" or from any kind of input, but much rather an emergent entity that derived from artificial biological and thus "evolutionary" rules.

> *My main objective in designing* Blind Watchmaker *was to reduce to the barest minimum the extent to which I designed biomorphs. I wanted as much as possible of the biology of biomorphs to emerge. All that I would design was the conditions – ideally very simple conditions – under which they might emerge. The process of emergence was to be evolution by the Darwinian process of random mutation followed by nonrandom survival. (Dawkins, 1988, p. 201, emphasis in original in italics)*

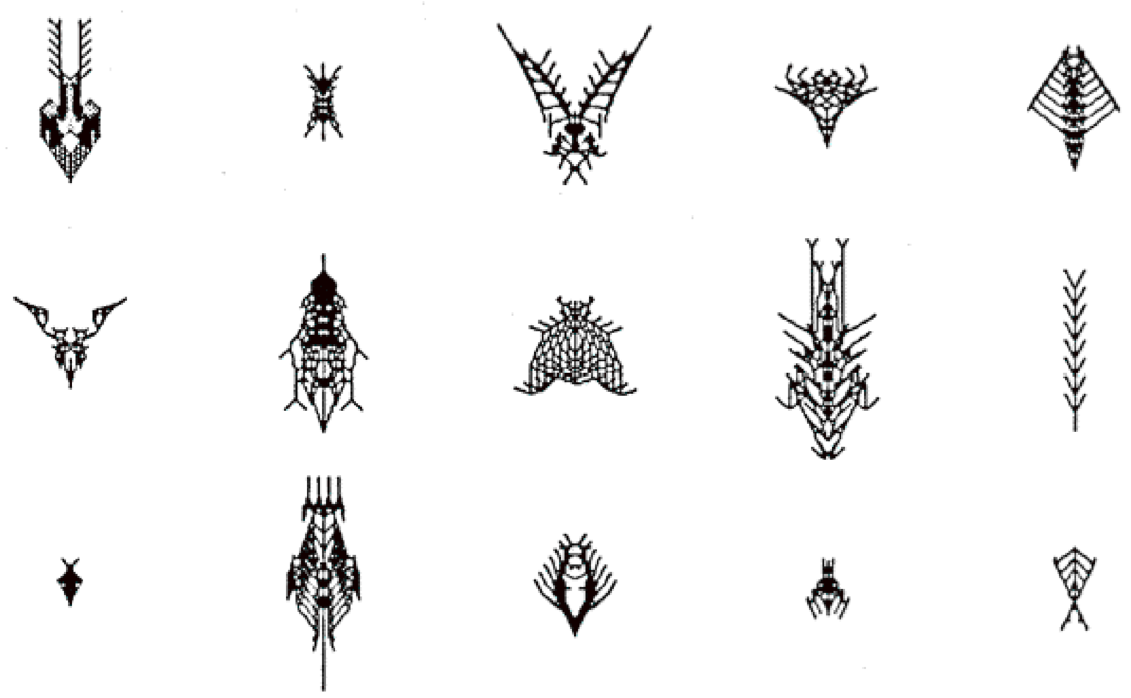
As explained in more detail in Dawkins' paper, what was used to represent the two-dimensional representations of these biomorphs was very specific: a Macintosh screen with the pixel dimensions of 340 x 250. Keep in mind, it was still the 1980s. This was important though, because it limits the possible phenotypes of these "creatures" to a total of 85000 pixels per frame. This is also the number of digital genes one of these creatures could have, while each genome represented by a single pixel could either be 1 or 0. For better understanding, think of each generated frame on the screen as being a collection of randomly assigned pixels with a value of either black or white, 1 or 0. The resulting sequence would look much like random static noise without making much sense or having some kind of decipherable patterns to it. Every new frame generated over time would enable the genome of the biomorph to mutate into its opposite state. So the pixels that were 0 in the first iteration could become 1 in the next "generation" of this specific biomorph. As we have already dipped into the wonderful waters of randomness combined with infinite iterations with more than just our toes, we can already project where this kind of approach is heading to:

> *It follows, therefore, that we in theory could "breed" any picture from a random starting pattern (Figure 2a) or, indeed, from any other picture, getting from, say, Winston Churchill to a Brontosaurus, by scanning every generation hopefully for slight resemblances to the target picture.*
>
> *But only in theory. In practice we'd be waiting till kingdom-come. This really would be a very naive way of writing development, and it would produce a very boring kind of artificial life. It is a kind of zero-order embryology, the kind of embryology we must improve upon. Our improvements will take the form of constraints. (Dawkins, 1988, p. 204)*

What differentiates so-called random pattern generations from real "artificial" evolution is, of course, some kind of selective process behind the scenes which determines whether the mutations of the digital genes makes "sense" within the simulation's constraints. This process is called "constrained embryology" and means that each generation has only a restricted set of phenotypes that can be generated while also stemming from a much smaller set of genes *(Dawkins, 1988)*. Although these new genes will also be much more powerful in how they can alter the biomorphs' evolution. Instead of only consisting of singe pixels they are now programmed to form more complex relationships with the shapes they are allowed to breed with, like, for example, lines with different mathematical properties instead of simple unrelated pixels. The goal here is to aim for an embryological process that produces phenotypes which are restricted in an biologically or aesthetically interesting way.

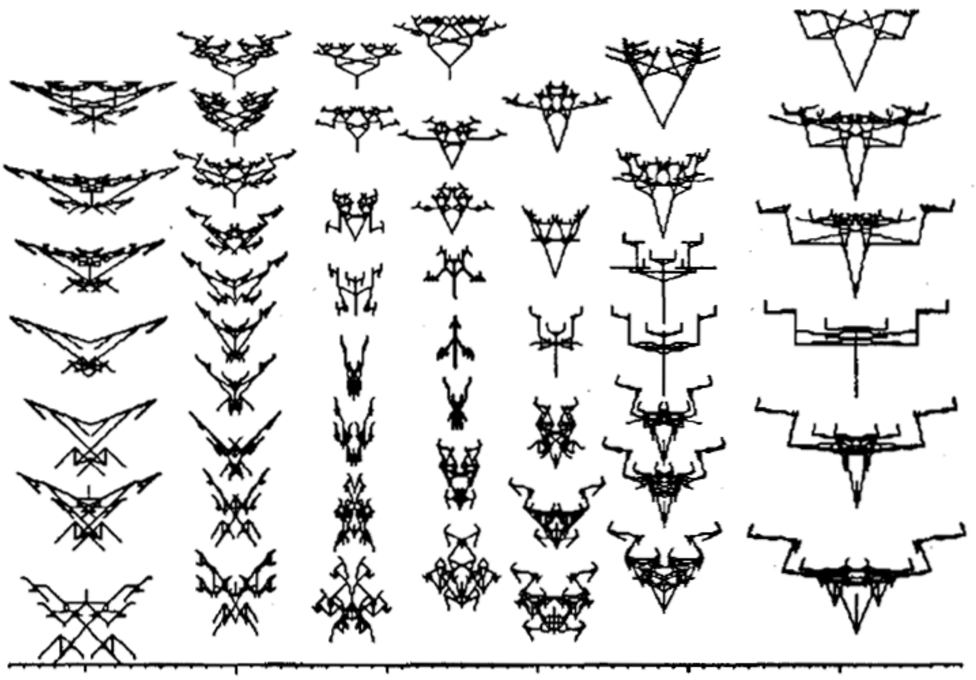
If we take a closer look on real-life biology, this also often means that the emerging patterns in organisms more often than not contain patterns of symmetry, either bilateral (mirrored) or radial *(Dawkins, 1988)*. Moreover, their is another very distinct property of the world around us, which we already have explored more in depth in a previous chapter. It is called recursiveness. By implementing more complex genes which are reproducing geometrically more complex patterns like lines, while at the same time adhering to meta-rulesets that favor symmetry and recursiveness, Biomorph Land accomplishes a degree of artificial evolution where its respective generated phenotypes already start to take on shapes very similar to what we are familiar with from real biological studies. To give a better understanding of how such constructs look like, figure 40 shows a few examples from Dawkins' program:

Figure 40, Phenotypes of "Biomorph Land" by Richard Dawkins



(*Segmented Biomorphs*, 2014)

Figure 40, Phenotypes of "Biomorph Land" by Richard Dawkins



(*Segmented Biomorphs*, 2024b)

I think of Biomorph Land as a digital and unintentionally artistic manifesto of the Darwinian process which sets out to prove that the possibilities of such an universal ruleset of natural selection are essentially open ended and can also result in something odd, like human beings. Literally "playing" god, in that case, is very beneficial, since for experiments like this, we can choose a digital biotope of pixels, rather than abuse conscious beings from the real-world. Moreover, we do not have to wait for nearly as long as it would take if we would like to demonstrate similar effects with real lifeforms. And time is certainly of essence in the field of artificial evolution and everything else that has emergent undecidable properties.

Another prominent example of an evolving system is the so-called *Langton's Ant*, named after Christopher Gale Langton, who is an American computer scientist considered a founder of the field called "artificial life" *(Murphy, 2023)*. He actually coined the term in the late 1980s when holding a workshop at the Los Alomos National Laboratory titled *Workshop on the Synthesis and Simulation of Living Systems*. Langton's Ant is a variation of a cellular automaton and thus is considered a "two dimensional universal Turing machine" *(Weisstein, n.d.-c)*. Therefore, it shares the same characteristics of Conway's game of life, meaning that one of the defining properties they have in common is a simple demonstration of the undecidability of the halting problem previously discussed. The "ant" itself is an artificial life form represented by a single pixel living on a grid of infinite pixels. The ant moves around the grid following only three very basic instructions for every iteration:

> *1. If the ant is on a black square, it turns right 90° and moves forward one unit.*
>
> *2. If the ant is on a white square, it turns left 90° and moves forward one unit.*
>
> *3. When the ant leaves a square, it inverts the color. (Weisstein, n. d.-c)*

The evolving pattern when running the simulation on an empty finite or infinite grid looks something like figure 41:

Figure 41, Demonstration of "Langtons's Ant" by Christopher Gale Langton

(*Langton's Ant*, 2024)

What it interesting about this cellular automaton is that the ant starts building erratic patterns for roughly the first circa 10,000 iterations before suddenly switching to building some kind of "highway" or tunnel pattern towards one edge of the pixel space. It does that every singe time the simulation is run *(Weisstein, n.d.-c)*. What is even more puzzling is the fact that no matter the starting pattern of the grid itself, meaning that if you would change the state of individual pixels on the grid a priori to some random pattern or a picture of either a Brontosaurus or Winston Churchill, the ant still defaults to building a tunnel after a certain period of time or amount of iterations. This poses the important question: Are there any initial states of the pixel grid that do not lead up to the eternal loop of tunnel building? So far, no exceptions have been found from any experiments on this matter whatsoever. This means that, up until now, there are theoretically infinite complex configurations of an infinite grid where the ant at some point in time or at some definite iteration throughout infinity always starts to build its highway towards any edge of an infinite grid *(Gajardo et al., 2002)*.

The defining sequence of the highway consists of 104 distinguished moves, which will then repeat from the beginning and will go on forever. But how do we know it will go on forever? The short answer here is that we actually do not know at all. Through all we know about the algorithm and the so called *Cohen-Kung Theorem*, which guarantees that the trajectory of Langton's ant is unbound *(Weisstein, n.d.-b)*, we come to the conclusion that the above described must be true, yet it is still unproven that it actually always will be true in any arbitrary configuration and thus is completely undecided. What I find particularly interesting about this example, besides its undecided nature of course, is the fact that there seems to be some kind of self-sorting mechanism at play. Even the plain instructions for the ant itself at first produce some kind of seemingly chaotic pattern before it defaults to the repeating tunnel sequence. The phenomenon is even more puzzling if the initial grid has been tampered with beforehand:

> *Most mathematicians who have studied the problem believe that there even is no general analytical method of predicting the position of the ant or of any such chaotic system after any given number of moves. It's behaviour can not be reduced to the rules that govern it. In the language of chaos theory, the pattern is a stable attractor for the system. (discovermaths, 2022)*

I consider this a wonderful metaphor of something that can be both chaotic and undecided, while at the same time demonstrating deterministic and harmonic properties. It is a box in a box in a box. I find it fascinating that one can rely upon some kind of order which is almost certain to be infused into the machine, even when it is in its most chaotic state imaginable. Even more so when the harmonic pattern resembling order is itself in a way chaotic, making it never quite sure when and if it will emerge at all, although as of yet, it never seems to disappoint. And even if the cycle is started anew, while leaving the previous iterations intact, amidst all the chaotic patterns emerging from harmonic tunnels intertwined with wild and random shapes of pixels, the ant is like a digital representation of nature itself. Unbothered and untamed, it always seems to find its way through an ever-changing landscape of shapes and chaos. Much like what we call "art" in a broader sense is capable of doing within our minds.

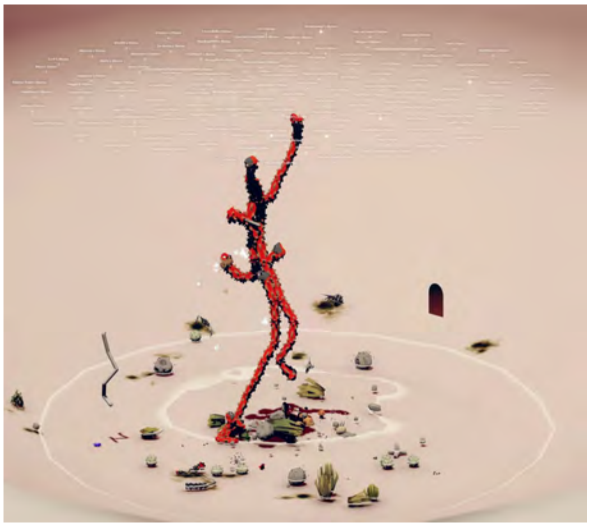
Are we BOB?

At this point, I want introduce Ian Cheng, a media artist currently based in New York who has dedicated his artistic career to simulations, artificial lifeforms and their respective capacity to interact and anticipate change inside ever-evolving systems. In his own words, he is interested in making "art with a nervous system" *(Pilar Corrias, n.d.)*. One of his contributions to the art world is called *BOB,* which stands for "Bag of Believes". BOB is an "AI-driven creature whose personality, body, and life script evolve across exhibitions." *(Cheng, n.d.-a)*

Figure 42 on the next page shows some pictures of BOB:

Figure 42, "BOB, Bag of Believes"
by Ian Cheng

(*BOB*, 2019)



(*BOB*, 2019b)

(*BOB*, 2019c)

(*BOB*, 2019e)

(*BOB*, 2019d)

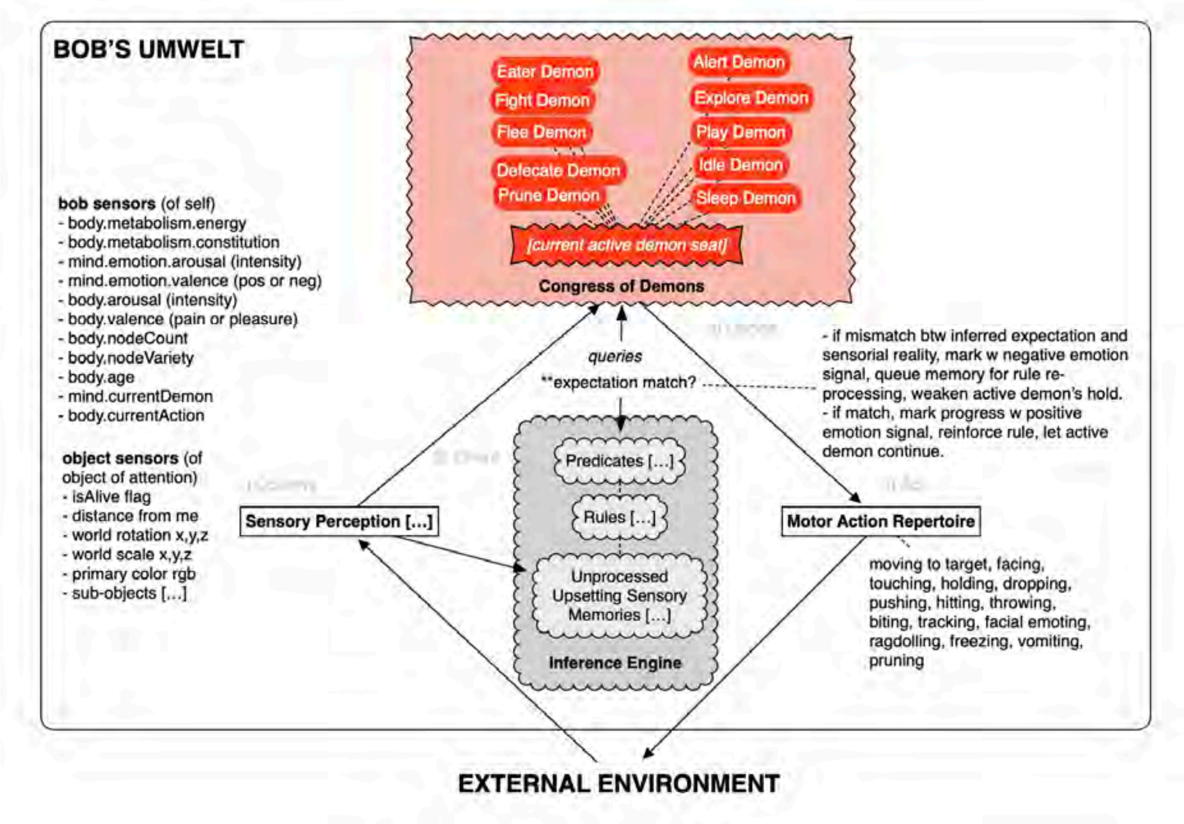
With this particular piece of digital art, Cheng wants to "use the idea of a creature as a kind of compositional space the way that a painter has a canvas as a compositional space, or a filmmaker has sets, cameras, special effects, actors as their compositional space." *(Serpentine Galleries, 2018).*

In other words, what he wants to achieve with BOB is to leave the confines of conventional compositional boundaries which normally are the medium through which artistic concepts are transported. While users still experience BOB through the medium of a screen, the creature named BOB itself is the analogy to that relative relationship between the viewer and the medium. BOB exists on a meta-level relative to the compositional space as an ever-evolving and adapting creature, incorporating his relation to the visitors into his own de facto actions, transforming the concept of BOB itself into an agent of its own conclusions rather than only being there to be observed strictly through the medium of the screen as a passive artwork:

> *I first became interested in making simulations when I felt a deep desire to see art that had a sense of aliveness and that could change over time and be a bit more out of control in terms of its authorship from me. More and more I've come to feel that I want to make art with a nervous system, and by that I mean art that can learn and react on its own and can react to you and me as a viewer. I think art with a nervous system starts to become something that feels sentient, that has its own agency and you would forgive it for not behaving. We expect art in a museum or an institution to behave perfectly as planned to the perfection of the artist and I am trying to both adapt myself personally and my work to be more pliable or adaptable to change and I figured by trying to make an artwork that can change, and hopefully change on its own, that you would have a relationship to it, the way that you have to any other living creature. (Serpentine Galleries, 2018)*

What I especially appreciate is Cheng's approach of creating art that is not "behaving" in a conventional way in respect to the artistic craft. It bears a strong resemblance to the art of Tinguely, which we have explored earlier. It is in the same way deterministically inconvenient as far as the artistic process goes. In both cases, one digital and the other analog, it is an artistically undecided endeavor. As we have seen in Tinguely's example, the approach of creating art that does not "perfectly behave" as expected in "a museum or institution" was not always as common a practice as it might seem today. It was - and still is - a radical idea. The concept that art itself is or can be an interruption of the artistic anticipation or relationship of the artwork per se and relative to the one who experiences it, is, in my view, a defining characteristic of the undecided nature deep within the the fabric of creativity and expression itself. So let us take a closer look on what BOB itself actually is by looking at a diagram of "Bob's Umwelt" in figure 43 *(Cheng, n.d.-c)*:

Figure 43, Diagram of "Bob's Umwelt" by Ian Chang



(BOB'S Umwelt, 2019)

In my interpretation, BOB can be seen as a Platonic meta-creature which lives in the space between expectation of how things should be and how things truly are. Cheng calls this state "Minimum Viable Sentience" *(Cheng, n.d.-c)*. As described further in his own portfolio, this state is achieved by intentionally making an agent like BOB "upset". What he means by "being upset" is the result of a system experiencing a mismatch of expectation vs. reality:

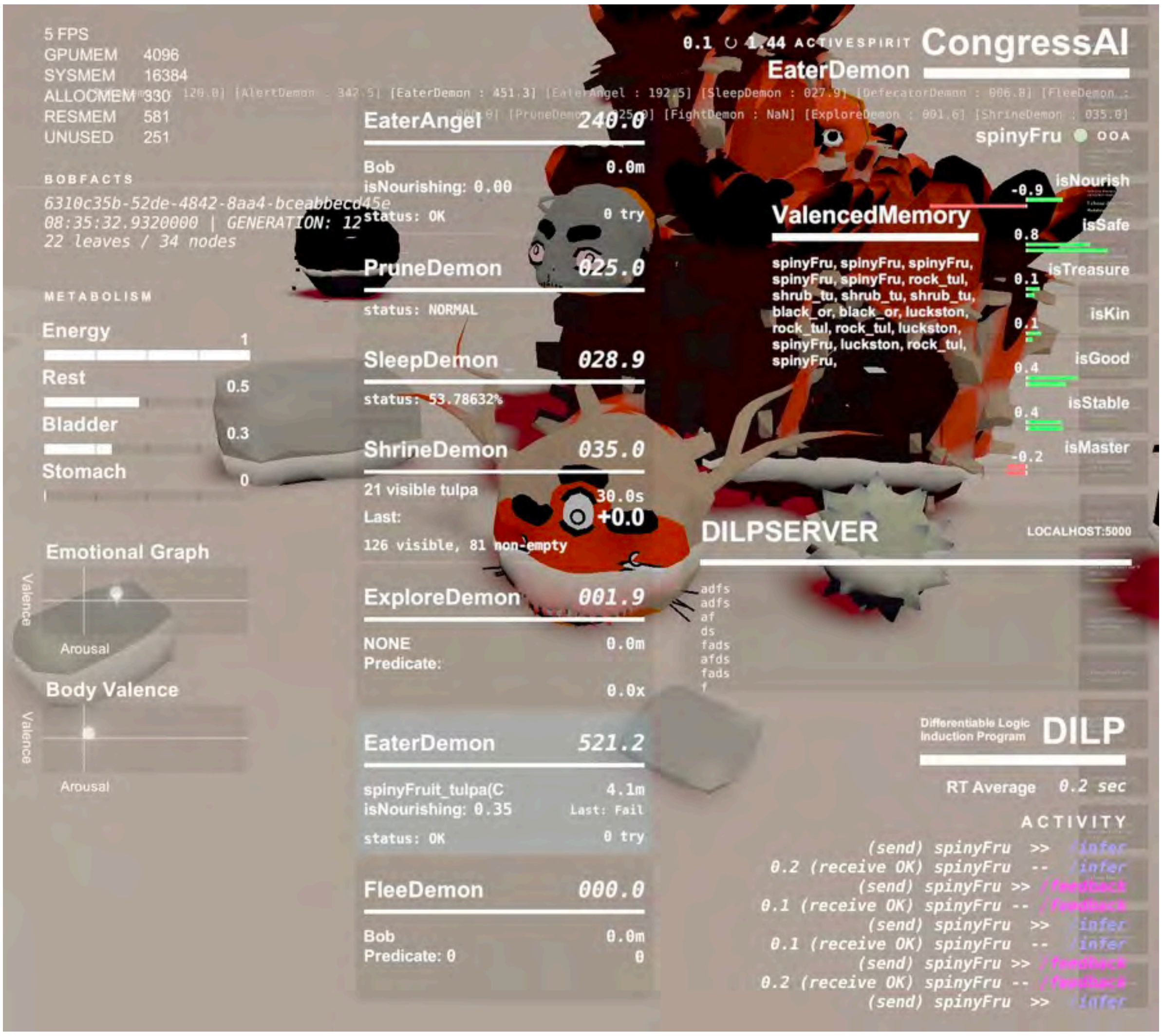
> *When a sentient agent is upset, it is experiencing a mismatch between its expectation of how it believes things to be and the reality of how things turn out. It manifests its beliefs into action, and they*

> *are not affirmed. The agent might double down and assert itself again upon reality. But if the mismatch persists, negative emotion accrues, signaling the lack of progress. The agent must undertake the work of updating its beliefs-–an energetically costly operation. This may destabilize further beliefs on which it is predicated, causing a chain reaction of cognitive upset. (Ian Cheng, n.d.-b)*

This then leads to a kind of recursive loop, a kind of algorithmic fractal behavior where, much like Langton's ant, at some point the pattern of chaotic loops of upset or mismatches are broken in favor of finding "a belief that might minimally reunify its upsetting experience with all of its other coherent representations of the world." *(Cheng, n.d.-b).* With each iteration, the agent might become more effective in successfully managing to integrate longer periods of minimal "upset" into its system. Thus Cheng calls it "self-legislating successfully" *(Cheng, n.d.-b)*. And according to him, "This is the achievement of a life lived sentiently." *(Cheng, n.d.-b).* Of course, BOB is only a digital creature comprising many different parameters augmented with different models of artificial intelligence. Its digital manifestations stem from the image of a snake which fractalizes into a branching tree-like structure with multiple body segments. This digital chimera is then able to constantly shift its point of gravity due to a locomotive, physics-based movement system inside its digital habitat. It also brings into existence different sensory parts on its body throughout its evolution in order to experience its immediate environment, as well as having the ability to sense inputs from beyond the fourth wall. This is complemented by a fixed set of predetermined skills, which enables BOB to navigate and manipulate its own path through the virtual space:

> *I developed a set of physical sensors for BOB: external sensors including pain receptors on each node; over-stretch detection between nodes; vision sensors that detect movement, color, shape, texture, basic composition of subcomponents; internal sensors including metabolic energy, constitutional integrity, and stomach and bladder capacity. Finally, I gave BOB a repertoire of basic actions: moving to a target, facing, touching, holding, dropping, pushing, hitting, throwing, biting, tracking, facial emoting, ragdolling, freezing, vomiting, pruning a body segment. (Cheng, n.d.-b)*

Figure 44, Backend of BOB by Ian Chang



(*Backend of BOB*, 2019)

While BOB's entire cognitive architecture would be too complex to fully explain in the scope of this thesis, I want to briefly summarize what it is capable to produce in terms of its emergent character. According to Cheng, he drew inspiration from Karl Sims' classical works as well as Richard Evans, an AI Scientist from the *Deep-Mind* project. Evans himself developed the concepts of designing computational systems with the ability of constructing and applying inherent beliefs from unstructured sensory inputs by looking at the insights of Immanuel Kant's work. Also, its internal programming and algorithms are structured in a way that was mainly inspired by the works of psychoanalyst Carl Jung and his notion that what we generally conceive as one single person might in reality be a collection of many different sub-personalities which he calls "demons", each having their own motivations and beliefs that constantly interfere with each other and are battling over our conscious dominion:

> *From these inspirations, I decided to center BOB's cognitive architecture on the relationship between desires and beliefs. Beliefs organize desires. Desires act on the world. The world affirms or upsets beliefs. (Cheng, n.d.-b)*

BOB is a "congress of demons" *(Cheng, n.d.-b)* who, as an ever-evolving system, are capable of constructing and deriving their own conclusions about the systems they exist in. It is not an artificial system that strives for perfection in terms of completing a task, it is more like an infant who tries to grow up while making predictions about what is waiting for them out there through the endless incoming stream of sensory data, reacting and adapting accordingly along the way.

> *Once even a few rules are established, it can then return inferences about any phenomena or object (a collection of sensed phenomena) from its constructed beliefs. An inference takes the form of a predicate with a confidence score. It essentially answers the question, 'what are all the qualities I can tell about what I'm looking at?' (Cheng, n.d.-b)*

So without hesitation, we can ask ourselves: Are we, too, some kind of BOB? Are we ourselves the miracle work or a self-imposed illusion of undecidable metapersonalities whose own cognitive architecture is stranded endlessly striving to optimize our own affirmations and upsets against what we want to believe? Maybe the world around us and the universe itself is nothing more but the physical constraint in which the compositional space of our existence takes place. Maybe we are constantly pressing our nose against the metaphorical window pane that is reality, trying to look through to the other side, trying to grasp if we are observed or if we ourself are the experiment that experiences itself. Is it possible that we are the original artwork and the art that we produce and conceive is only another fractal iteration of us as artists by existential means? The answer to these questions is floating in the vastness of an undecided and perpetual cosmos of being who we are and when we are. Thus, existence itself, is art.

uchū

Figure 45, "uchū" Phase 0 (title screen)

In the last part of this journey through the vastness of the undecided and unknown, I want to offer you a seat as a pilot to experience the underlying concepts of my thesis in my artwork called *uchū*. The word is derived from a Japanese kanji which means universe, cosmos, heaven and earth, space and time in the context of infinity. *uchū* is an audiovisual moment providing the user with a perspective of our relation to the undecided and invites you to explore and meditate, to stop and be still for one particular moment in time and reflect on everything that eludes us so effectively within our mind. It serves as a reminiscence of everything I have touched on in this thesis. You become the creator of that moment, which itself will be forever lost and never again repeated after it has faded back into the digital void.

How does it work? *uchū* is essentially a generative system that works on the principles of true randomness in both its visual components and in the synthesis of its soundscape. At the core of its visual representation are so-called geometric "supershapes", which are a way to simulate and generate natural looking polygons or complex organic shapes described by an equation and different input parameters. Its soundscape is composed in real time via a virtual rack of different modular synthesis modules working together in unison to produce a virtually endless and non-repeating soundtrack that feeds of the true randomness of its underlying virtual "DNA". See figure 46 below for a screenshot of the virtual modular synthesizer rack of *uchū*:

Figure 46, Virtual modular synthesizer rack of uchū

The virtual DNA of these soundscapes and supershapes is reprogrammed every time the cycle of the system is reiterated by creating a sequence of numbers injected into it by true randomness in the form of quantum measurements and user interaction. The installation itself consists of 4 main tangible components:

- A visual representation of the system which can either be any screen or a wall projection

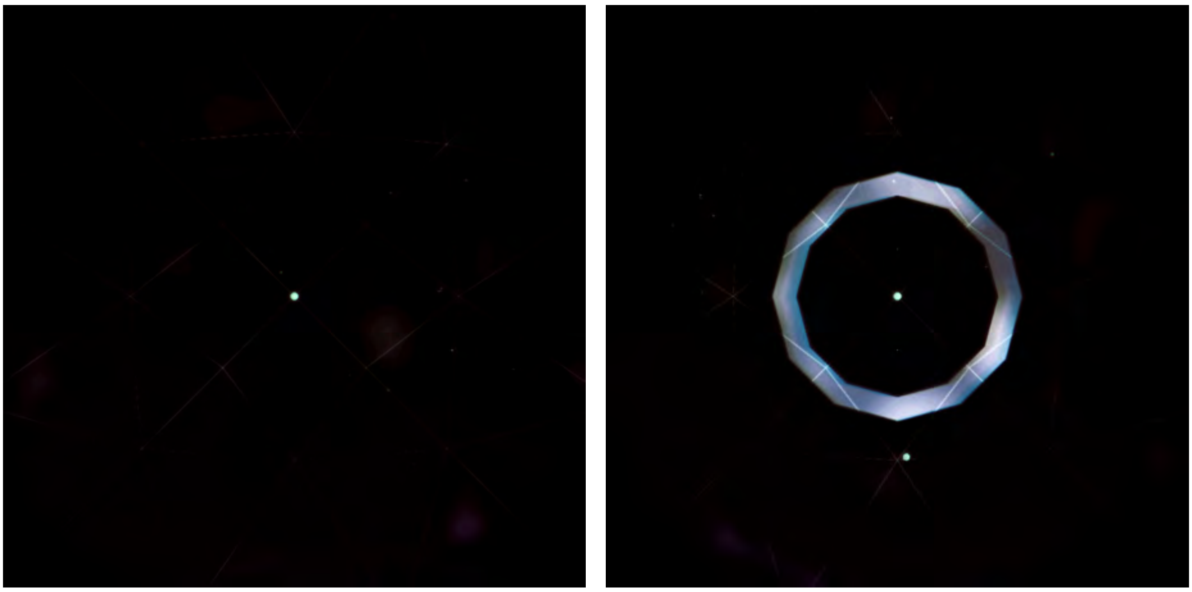
- An auditory representation of the system which has to be fed into a sound system like speakers or headphones in order for the user to experience it

- A visual input device like a webcam which is constantly trying to detect and track three different parts of a human being, namely the face and each individual hand

- An auditory input device like a microphone

In order to start a cycle, an "uchū-naut" has to stand in front of the screen and webcam. As long as the person has a face that closely resembles a human being, a circular shape will appear on the screen, indicating that the system has begun to notice their presence while also serving as a symbol for infinity. This shape also reacts to audio input and starts to vibrate in order to encourage further exploration and to indicate that something is being processed in real time by the simulation. See figure 47 below for screenshots of that particular state (phase1):

Figure 47, "uchū" Phase 1



Invisible to the user, the tracking data of his face and the surrounding noise (i. e. the audiovisual feedback), is also fed back into the algorithm which then starts to sample true random values through a web API. This particular API is providing random numbers based on quantum fluctuations in the vacuum of space in real time from the laboratory of the Australian National University:

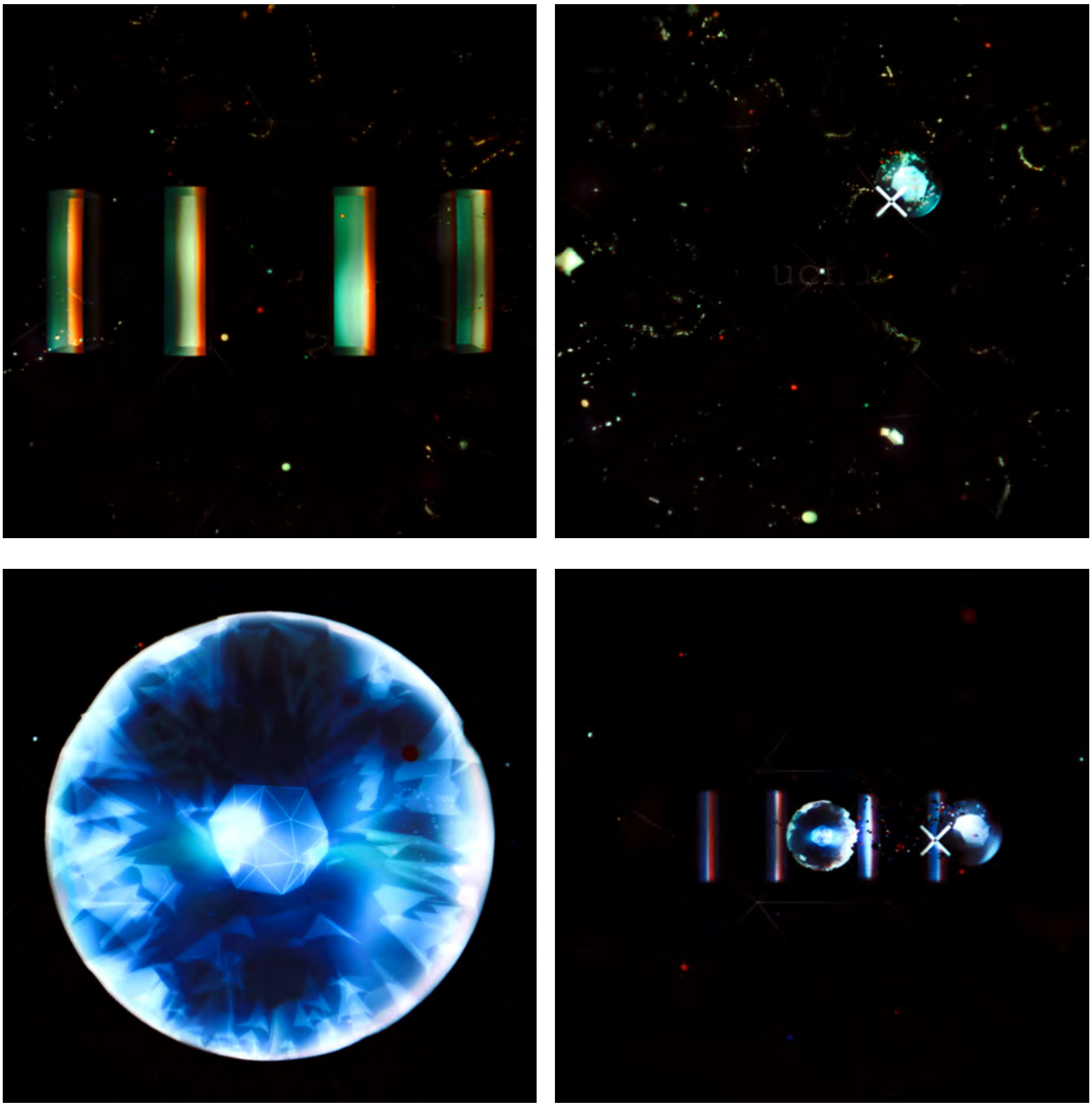
> *The random numbers are generated in real-time in our lab by measuring the quantum fluctuations of the vacuum. The vacuum is described very differently in quantum physics and classical physics. In classical physics, a vacuum is considered as a space that is empty, devoid of matter or photons. Quantum physics however says that same space resembles a sea of virtual particles continuously appearing and disappearing. This is a prediction of quantum mechanics and can be measured and even sometimes utilised, such as in this service. By carefully measuring these vacuum fluctuations, we are able to generate ultra-high bandwidth random numbers. This means our random numbers are truly random, as guaranteed by the laws of quantum mechanics.* (*ANU QRNG*, n.d.)

This sampled random sequence, combined with the algorithmic feedback of the user interaction then starts to form the digital composition of every simulation cycle. The sequence is fed into a midi sequencer as well as into the parameters that define the construction of the geometric supershapes. To summarize, this true RNG sequence is the basic building block of every element within *uchū*. It is, so to say, its very own, ever-changing and thus undecidable digital DNA. It determines the outcome of the audiovisual expression of *uchū*, while also defining the basic physical effects and properties of the cosmos these supershapes and sounds live in.

As of this initial moment, only a digital void on the screen is visible which is representative of the simulation's own digital space-time continuum. What happens next, after a person has been detected in front of *uchū*, is that the system charges up all its internal parameters into a set of rules that will govern the momentary cycle it lives in. It presents the user with different geometric forms that appear on the screen, and the coordinates of the right hand are mirrored into the digital microcosmos in form of a translucent sphere, called the "wanderer", which contains a singular supershape of this one particular iteration as a reference while at the same time being a notion to our place in the context of our own *uchū*, which we call the universe. After the wanderer has appeared, one is then able to move the sphere around in virtual space by moving their right hand around and collide the wanderer with the other geometric shapes evolving in the simulation. Each collision with any of these objects will further randomize and alter the digital DNA of its iteration, while only a visual hint of these transformations remains visible to the human eye.

After a random amount of time, all the programming of the DNA is complete and will be frozen internally. This state of the machine is represented as a temporary bubble, containing the randomly defined supershape as a representation of the now finished parametrization. It is now time to initialize the final phase of *uchū*.

Figure 48, "uchū" Phase 2



As soon as an uchū-naut is directing the wanderer into this bubble, which, as already stated, is the symbolic placeholder of everything it contains, an internal countdown is started to the externalized simulation which I will call "little bang" for the sake of the very obvious analogy to our own universe's presumed beginning. When the countdown reaches zero, this bubble bursts into all its smaller parts that were defined by its digital DNA, and gives birth to a single iteration of a microcosmos which lives inside a randomly predefined time and space, governed by randomly predefined rules. Inside this continuum, there are different physical force fields that arrange the newly born supershapes into truly random patterns, give them a spectrum of random colors, and apply forces of direction and movement while the supershapes themselves are, every single time, constructed out of truly randomly generated configurations of polygons. They all inherit pieces of the collective digital DNA, including their physical attributes and lifespan. The soundscape also follows these principles, as layers of randomly sequenced synthesis are added procedurally to the experience and also react to the different states of *uchū*.

In the final phase of the simulation (phase 3), the uchū-naut now takes on the role of an active observer and is able to navigate and explore this machine-dreamscape with the wanderer as their avatar. They can move through the streams and patterns of entities and influence their movement and trajectory by colliding with them, and in the process put their very own and unique signature into the evolving system. The soundscape also reacts to these random encounters and changes with every interaction. At the final stage of the iteration, the user can also use their left hand in order to navigate the perspective of *uchū*. They can steer the point of view freely in any direction and also zoom out into infinity up to a point where the whole spectacle fades into almost a singular point on screen. During the whole time *uchū* evolves, there is no inherent method to save or reproduce any exact moment in any way. It will be forever lost after it took place, and is only there to be observed in this personal moment of interacting with *uchū*. This is intentional, of course, since I want this to be a temporary and immersive experience that can not be conserved in any way. Every interaction and iteration of the simulation will occur only exactly once and can never be reproduced again in its exact form.

See figure 49 on the next 3 pages for some impressions of such an iteration:

Figure 49, "uchū" Phase 3

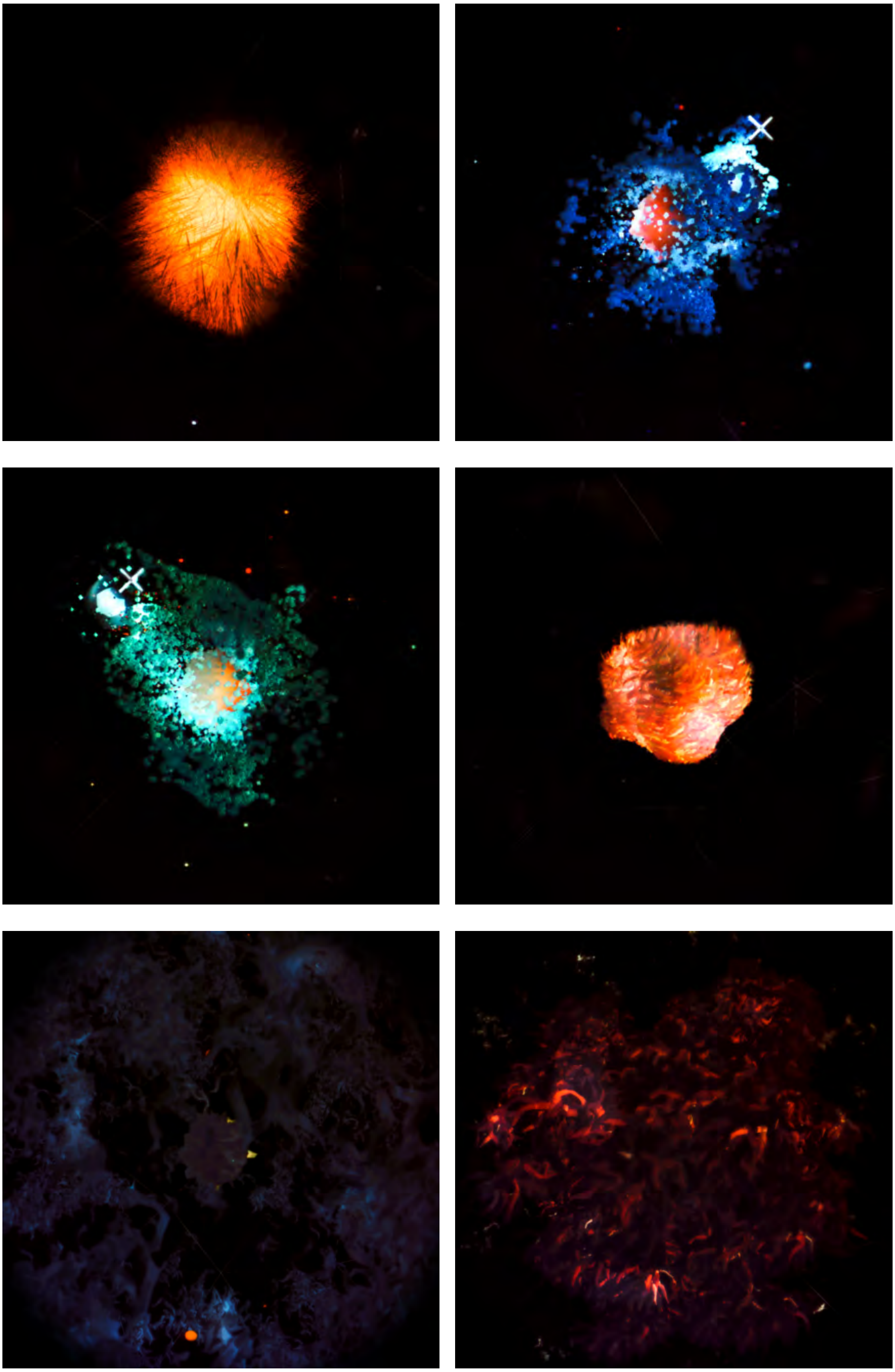


Figure 49, "uchū" Phase 3

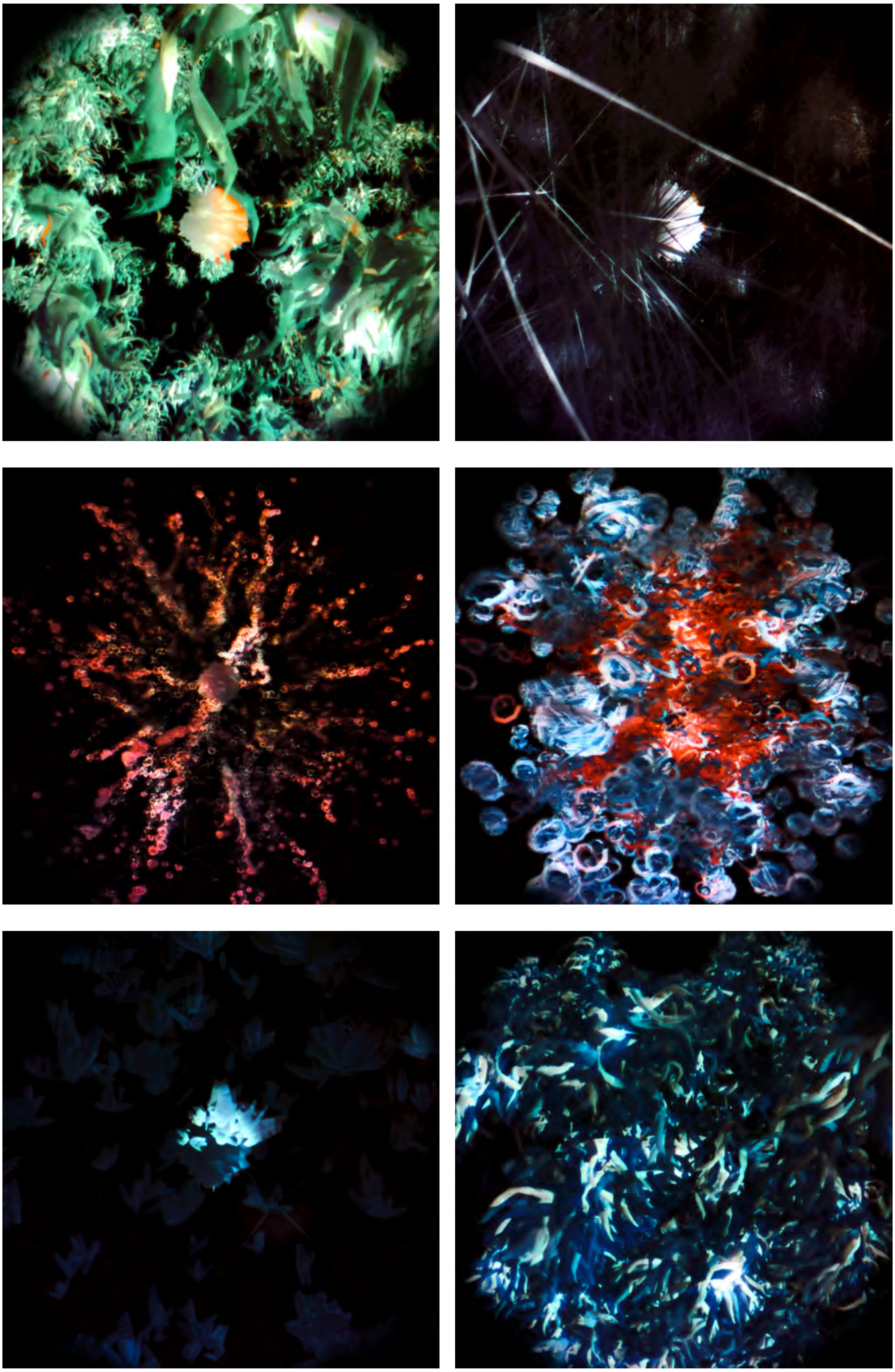
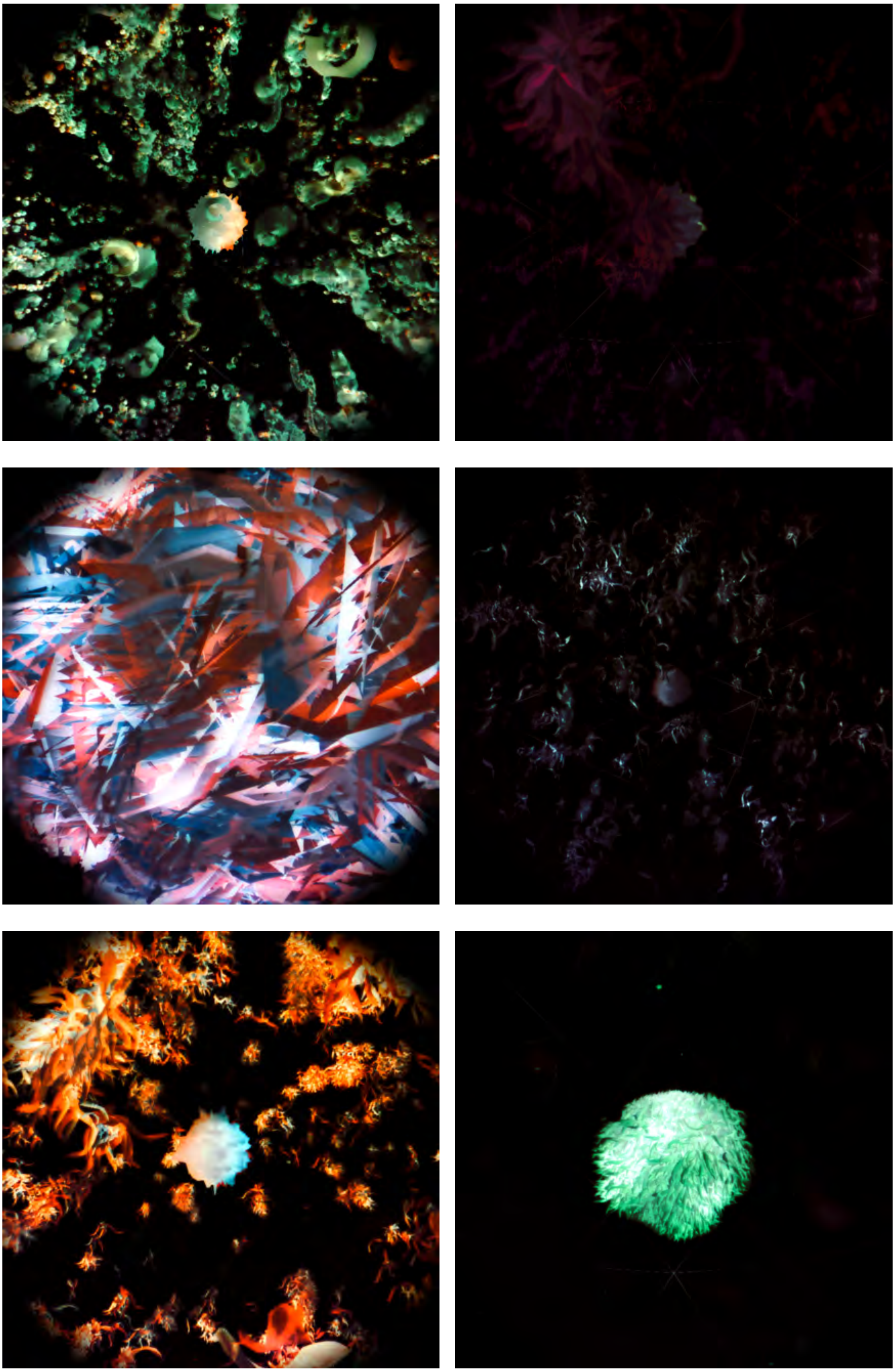


Figure 49, "uchū" Phase 3



Moreover, the outcome of the simulation in the context of how the system evolves as a whole itself is completely undecided, which means that the general iterations of one such circle inside this cyberspace is not known and could theoretically go on forever. Although, for the purpose of demonstrating the concept of *uchū* in the constraints of my thesis, I implemented a safe switch to ensure that one such iteration of *uchū* is finite and thus can be repeated without having to potentially wait an eternity before the cycle (potentially never) ends.

Of course this is only theoretically true, since the Turing effect of the simulation would only be valid if the computer *uchū* is running on would also at the same time "live" forever and meet all the requirements we have explored in this thesis for it to be truly undecided. That is why I consider *uchū* more of a sculpture and a place to meditate around these concepts rather than any kind of actual experiment or attempt to study or prove the underlying principles of my thesis. Also, as demonstrated in this thesis, proving that anything is undecidable can only be undecidable at best, and thus would be a futile effort. And while all of the above seems very technical and boring in an artistic context, most of *uchū*'s inner workings are well hidden from the beholders' eyes and senses. It is an attempt to turn all those ideas into an artistic manifestation that invites people to playfully explore an audiovisual microcosmos and light a little fire in their minds, maybe inspiring them to think about our place within the undecided nature of our reality. *uchū* has no rules in terms of its experience, only in terms of its technical inner workings. It bridges the gap between the playfulness of art and the scientific processes that govern our understanding of our universe, while giving a friendly and welcoming wink to all the concepts it encompasses.

"Embrace the undecided and explore your own inner *uchū*", is what its message shall be.

What dreams are made of (Conclusion)

As life unfolds, everyone finds themselves fully immersed in a dream of their own unique realities. We are challenged from within to linger, confront and contemplate the vastness of this conscious space we call the human experience. And from the undergrowth of that particular vantage point, we can witness the sparks of wisdom emerging from the void for a fleeting moment in time. Being human means to embrace and follow these beacons of light for a chance to engage with ourselves in retrospection and to reflect upon moments which make up the past and are already decided. This endeavor is the essence of defining the present as well as weaving an illusion of what the future might entail for us. We experience life itself as a great uncertainty. It is in that labyrinth of meaning that we often find ourselves being led astray and we keep struggling to make sense of our existence when venturing deep into our own minds, only to ask the questions that we know cannot be answered.

Our future, as well as the very fabric of the universe as we know it, rests upon the pillars of undecidability. This absence of certainty becomes the paradox of existence itself. We have to acknowledge that we cannot fully know something which means to also confront the reality that we may never truly know anything at all. Never knowing means being in doubt; in doubt with all that is, that ever was, or may come. Maybe it is this discrepancy that fuels us in our endeavors of finding a deeper meaning and fully grasping the fortunate dilemma of our existence. Human curiosity and longing for meaning lie at the very foundation of all creativity. What we consider art is a contextualization of our own insecurities and curiosity about our collective experiences. Art is the language of non-deterministic expression: It creates a vehicle to journey on, towards the frontiers of what we seek to discover about ourselves and the world that surrounds us.

What remains is to be humble in the context of everything that is. In this thesis, I have attempted to show the implications of the nature of undecidable systems throughout artistic and scientific history and their profound impact on our interpretations of subjective realities. I have shown how we are often not aware of how things really are. From Ancient Greek philosophical ideas such as *Plato's Cave* and the early beginnings of geometry like Euclid's *Elements* towards past and present examples of pop culture such as *Star Trek* or Douglas Adams' *The Hitchhiker's Guide to the Galaxy* and down to modern generative computer art like Herbert Franke's *Cellular Automata*, the generative *Biomorphs* of Richard Dawkins and rising technologies like artificial intelligence. In all of those examples, undecidability is undoubtedly a fixed constant. It manifests itself in many ways, often hidden beneath the surface of the complex properties of human creation while at the same time it can also be found at the intersections of art and science. This inspired artists like Cornelis Escher, Jean Tinguely, Jackson Pollock, Pablo Picasso and the whole Cubist movement, just to name a few. It also had a great impact on scientists who became artists in exploring these undecidable properties of reality in a creative way like Martin Krzywinski's *Approximations of Pi* or Benoit Mandelbrot, amongst others.

Long before the term "art" was even coined, humans have been artists. Through art, we communicate with each other and with the cosmos. I have explored how art can give context and meaning through its inextricably attached undecidable properties that make it an expression of undecidability itself. It is a creative and often abstract reference, trying to dissect and combine the smallest possible parts and pieces that make up our perceivable world. It opens up the possibility of contextualizing reality for the sake of gaining a better understanding of one another, or at the very least for serving as a place in space and time for reflection and introspection. It helps us to keep a little warmth in the vastness of uncertainty.

This thesis is an attempt to set sail and go full speed ahead into the unknown. It serves as a map to explore and navigate the seas of undecidability. Because, as I see it and as may be concluded from my research, everything could ultimately be a matter that is inherently undecided. Even the construct we label "art" shares these undecidable properties of reality which are inseparably interwoven with its manifold manifestations.

And if everything is undecided, everything is art.

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